1 OF 1
AD-
A078160

END
DATE
FILMED
1-80
DDC

AD A078160

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Methodology for Evaluating In-Lake Effects Resulting from Phosphorus Management In the Lake Erie Basin.		5. TYPE OF REPORT & PERIOD COVERED Final rept.
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Ralph R. Rumer, Jr.		8. CONTRACT OR GRANT NUMBER(s) DACW49-78-C-0005
9. PERFORMING ORGANIZATION NAME AND ADDRESS State University of NY at Buffalo Civil Engineering Dept. Buffalo, NY		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Water Quality Section NCBED-HQ U. S. Army Engineer District, Buffalo 1776 Niagara St., Buffalo, NY 14207		12. REPORT DATE July 78
		13. NUMBER OF PAGES 71
14. MONITORING AGENCY NAME & ADDRESS(If different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approval for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Copies are available from Technical Information Service, Springfield, VA 22161

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Lake Erie
Phosphorus
Water Quality

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

ABSTRACT

This report outlines the knowledge of the phosphorus budget in Lake Erie and describes a long-term total phosphorus model that has been used to simulate the phosphorus dynamics in the lake. The model incorporates the input of phosphorus to the lake from the drainage basin, exchange of phosphorus at the sediment-water interface and export of phosphorus from the lake by river outflow. Using available data and assuming equilibrium, the model was calibrated and used to project future in-lake phosphorus concentrations for a recommended objective loading of 11,000 metric tons of total phosphorus. The findings indicate that management of nonpoint, as well as point sources, will be required to meet the loading objective. The model output also enables the projection that the trophic status of the central basin of Lake Erie will be significantly improved (mesotrophic) if the recommended objective loading is obtained.

Accession For

NTIS GRA&I
DDC TAB
Unannounced
Justification_____

By_____

Distribution/

Availability Codes

Dist.	Avail and/or special
A	

D D C
RECEIVED
DEC 12 1979
D
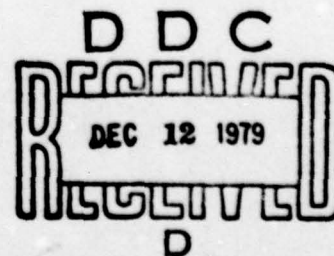
METHODOLOGY FOR EVALUATING

IN-LAKE EFFECT RESULTING

FROM PHOSPHORUS MANAGEMENT

IN THE LAKE ERIE DRAINAGE BASIN

by

Ralph R. Rumer, Jr.

July, 1978

D D C

RECEIVED

DEC 12 1979

D

ABSTRACT

This report outlines the knowledge of the phosphorus budget in Lake Erie and describes a long-term total phosphorus model that has been used to simulate the phosphorus dynamics in the lake. The model incorporates the input of phosphorus to the lake from the drainage basin, exchange of phosphorus at the sediment-water interface and export of phosphorus from the lake by river outflow. Using available data and assuming equilibrium, the model was calibrated and used to project future in-lake phosphorus concentrations for a recommended objective loading of 11,000 metric tons of total phosphorus. The findings indicate that management of non-point, as well as point sources, will be required to meet the loading objective. The model output also enables the projection that the trophic status of the central basin of Lake Erie will be significantly improved (mesotrophic) if the recommended objective loading is obtained.

TABLE OF CONTENTS

INTRODUCTION

The historical record documents the decline of commercial fishing
in Lake Erie beginning in the late 1920's ("Lake Erie Environmental
Summary, 1963-64," U.S. Dept. of Interior, FWPCA, May, 1968). From
that date until the present there have been significant changes in
the species composition of fish in Lake Erie. To some extent, these
changes reflect the impact of pollutant loading to the lake and its
tributaries and the accompanying nutrient enrichment of the lake.
This impact has manifested itself through increasing levels of
biological productivity and seasonal occurrences of oxygen depletion
in some parts of the lake (Burns and Ross, 1972; Great Lakes Water
Quality, Appendix B, IJC, 1976). Taste and odor problems in public
drinking water supplies taken from the lake are associated with this
biological productivity. Other related problems include the loss of
important spawning sites and unsightly accumulations of floating algae
in the nearshore areas and on beaches (Livermore and Wunderlich, 1969).

Many of the adverse impacts mentioned above occurred gradually
and were not a priori accepted as a necessary outgrowth of man's
activities in the drainage basin. It has taken some time to unravel
the cause and effect relationships that have contributed to the
observed adverse impacts. Although there is still much to be learned
about the Lake Erie ecosystem, the state of knowledge has advanced
to the stage where many of these cause and effect relationships are

understood, quantifiable and can be expressed mathematically.

In the very early stages of the Lake Erie Wastewater Management Study, an inventory and evaluation was make of methodologies that could be used to predict the consequences of alternative wastewater management programs for the Lake Erie basin (Tetra Tech Report No. TC-413, October, 1974). The results of that review indicated that evaluation methodologies were available that could be used in the study. At the outset, it was apparent that a choice would have to be made as to what parameters should be modeled and what level of model sophistication would be appropriate. Eventually the study staff selected total phosphorus as the most significant parameter in terms of the problem area being addressed; i.e., eutrophication of the Lake Erie waterbody. This report summarizes several of the evaluation methodologies (or models) available or under development which might have been used to aid in the assessment of the possible in-lake effects that would result from the implementation of specific management strategies in the drainage basin. The methodology selected is described, its application illustrated, and the significance of the model projections are discussed.

Eutrophication

Hutchinson (1969) has pointed out that the term "eutrophic" originated in 1907 from consideration of the nutrient conditions or chemical nature of soil solution in bogs. In 1919, the term was introduced into limnology and has come to be widely accepted as describing a lake rich in nutrients. The term "nutrients" refers

to the chemical substances essential to the growth of plants. In
lakes, an important group of plants are the algae, or phytoplankton.
Phytoplankton require light for growth and carry out the photosynthetic
reaction utilized by all green plants. The macro nutrients important
to the growth of phytoplankton include carbon, nitrogen, and phosphorus
(Mitchell, 1974). Plant growth depends on the availability of these
and other nutrients. Thus, it would be expected that a nutrient-rich
lake would be abundant with phytoplankton growth, assuming that other
environmental factors (e.g., sunlight and water temperature) were
conducive to such growth. Thus, the excessive biological productivity
characteristic of eutrophic lakes begins with the primary producers,
the photosynthesizing phytoplankton.

Although the term "eutrophic" first came into use as a descriptor
for a nutrient-rich water, it is now recognized that a waterbody and
its drainage basin must be viewed as a coupled trophic system. In this
sense, a waterbody evolves toward a state of "trophic equilibrium" with
its drainage basin which is the source of most of the nutrients and
other constituents that are deposited in the waterbody (Hutchinson,1969).
Waterbodies that are nutrient poor are classified as oligotrophic. The
trophic status intermediate between oligotrophic and eutrophic is termed
mesotrophic.

The time scales associated with the trophic evolution of a water-
body are generally very long. However, high intensity agricultural
development, growth of human populations, urbanization and industrial-
ization can proceed very rapidly in a drainage basin. When this is the
case, the time scale for trophic evolution can be greatly shortened.

This latter situation is often referred to as accelerated cultural eutrophication. Strategies for dealing with this situation may vary, depending on the size of the waterbody and the complexity of the activities in the drainage basin. For large waterbodies and drainage basins, (e.g., Lake Erie) development of remedial strategies requires that the sources of nutrients in the drainage basin be identified and controlled.

Since photosynthesizing phytoplankton are the essential link between the sun and other life forms that cannot directly utilize the energy of the sun, it would be expected that abundant growths of phytoplankton would have repercussions in the aquatic food chain as well as affecting the chemical and physical equilibrium of the water-body. Dead and dying algal mass settle to the sediments where decay takes place through bacterial decomposition. Oxygen is consumed by this aerobic decay process and, in waterbodies where vertical mixing is inhibited (because of seasonal stratification), depletion of dissolved oxygen can stress other life forms that inhabit the bottom waters. This stress in turn can contribute to changes in the species composition and in fish populations.

This photosynthetic production of algal biomass and subsequent bacterial decomposition (or destruction) of the organic material produced is paralleled by a production and consumption of oxygen (Stumm and Stumm-Zollinger, 1972). In a balanced ecosystem, this cycle produces a constant surplus of oxygen. In a stratified lake, the production of oxygen occurs primarily in the upper waters (the

5
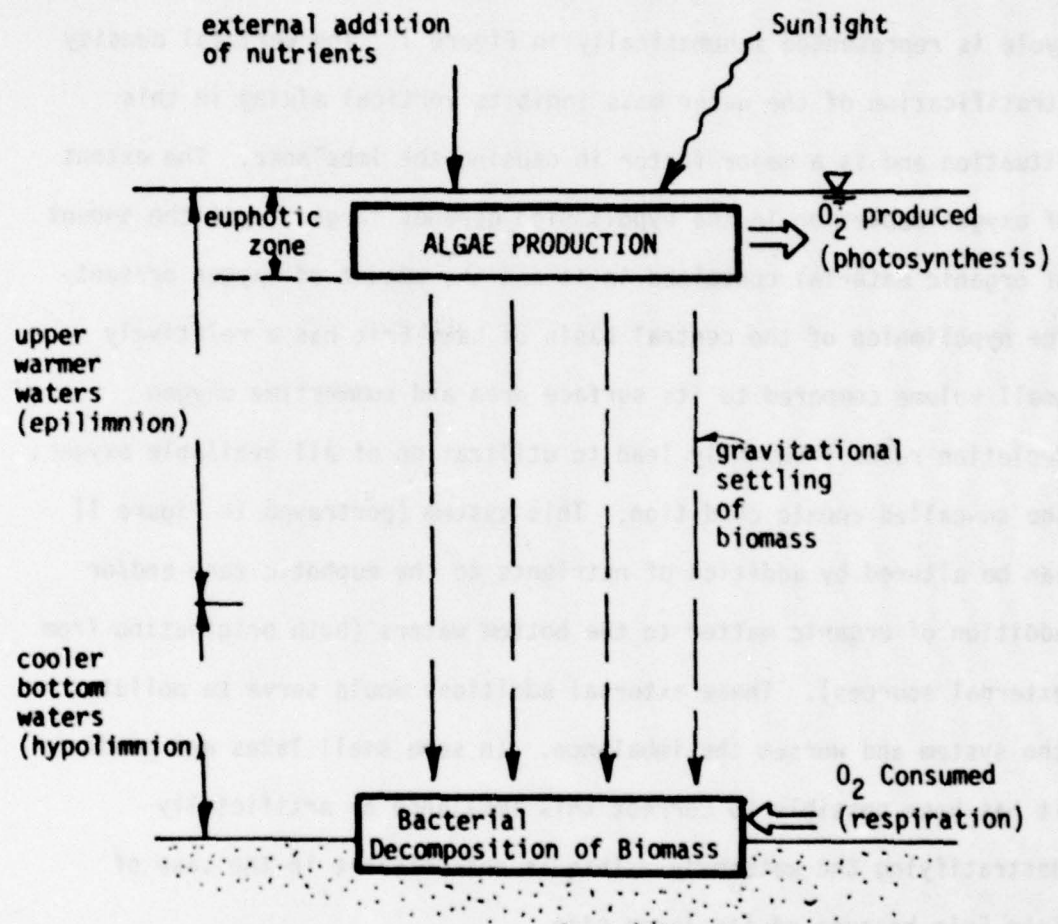
Figure 1 Photosynthetic Production and Bacterial Consumption of Oxygen
in a Stratified Lake

6

euphotic zone of the epilimnion) while the consumption of oxygen
occurs primarily in the bottom waters (the hypolimnion). The
separation of these two interactive processes leads to an imbalance of
the ecosystem and a depletion of oxygen in the bottom waters. This
cycle is represented schematically in Figure 1. The vertical density
stratification of the water mass inhibits vertical mixing in this
situation and is a major factor in causing the imbalance. The extent
of oxygen depletion in the hypolimnion depends largely upon the amount
of organic material contained in it and the amount of oxygen present.
The hypolimnion of the central basin of Lake Erie has a relatively
small volume compared to its surface area and summertime oxygen
depletion rates frequently lead to utilization of all available oxygen;
the so-called anoxic condition. This system (portrayed in Figure 1)
can be altered by addition of nutrients to the euphotic zone and/or
addition of organic matter to the bottom waters (both originating from
external sources). These external additions would serve to pollute
the system and worsen the imbalance. In some small lakes and ponds
it has been possible to correct this imbalance by artificially
destratifying the waterbody. This is not possible in the case of
Lake Erie because of its large size.

Role of Nutrients

Nutrients are cycled through the system portrayed in Figure 1.
Biologically available forms of the nutrients are utilized in the
production of biomass. During the decomposition process, bacteria
mineralize the organic forms of the nutrients into inorganic forms
and thus return the nutrients to a biologically available form so

the cycle can be repeated. The regenerated nutrients can be brought to the upper waters through intermittent mixing processes across the density interface that separates the epilimnion and hypolimnion, by wind-driven upwelling currents near shorelines, or during the seasonal overturning of the entire waterbody when no stratification is present.

When nutrients are viewed as essential ingredients to the photosynthetic energy conversion process, the concept of trophic status is seen to be an indicator of solar energy conversion efficiency. Oligotrophic lakes would be the least efficient and eutrophic (or hypereutrophic) lakes would be the most efficient. Mesotrophic lakes would have intermediate efficiencies. Vollenweider (1971) has discussed the difficulty of establishing a satisfactory trophic status classification system for lakes. Parameters such as phytoplankton density, chlorophyll content, biomass production or carbon assimilation are important indicators but have proved unsatisfactory because of the variability between lake drainage basin systems. Vollenweider reviewed available data on lake water chemistry and suggested a preliminary trophic classification method based on unit surface area loading of total nitrogen. He cautioned that this method could only be applied to situations where the ratio of nitrogen to phosphorus inputs was higher than the nitrogen to phosphorus ratio found in phytoplankton. Using the reported molecular formula for phytoplankton cell material (i.e., $C_{106} H_{263} O_{110} N_{16} P$), the computed ratio of nitrogen to phosphorus is 7.24. More conservative estimates give the ratio as high as 14 (Nat. Eutroph. Survey, 1974).

The Importance of Phosphorus

Although phosphorus has been selected as the primary nutrient
of concern, there is no conclusive proof that it is the limiting
nutrient at all times and at all places in Lake Erie. However, the
preponderance of evidence suggests that it most often is the key
nutrient (Stumm and Stumm-Zollinger, 1972). Certainly, the reduction
of phosphorus available for phytoplankton growth is an appropriate
management goal if it will result in reducing phytoplankton growth.
In-lake measurements indicate that available nitrogen, the other
important macronutrient, is about twenty times as great as available
phosphorus (Burns, 1976). Since the ratio of nitrogen to phosphorus
in phytoplankton cell material is conservatively estimated to be
less than fourteen (National Eutrophication Survey, 1974), this is
further justification that phosphorus may be the limiting nutrient in
Lake Erie. Hutchinson (1957) has discussed the phosphorus cycle in
lakes and stated that "...(phosphorus) is in many ways the element
most important to the ecologist, since it is most likely to be
deficient..." Sager (1976) in a review of lake eutrophication also
underlined the relative importance of phosphorus. The reduction
of phosphorus inputs to Lake Erie remains as an essential part of
the strategy for improving water quality as outlined in the Canada-
U.S. Great Lakes Water Quality Agreement.

PHOSPHORUS LOADING TO LAKE ERIE

Phosphorus enters the surface of Lake Erie from the atmosphere, at the lake boundaries from shoreline erosion of phosphorus bearing sediments, and in the tributary inflows that drain the watershed. The potential export of phosphorus into the lake from sources in the drainage basin is great. Because of Lake Erie's morphometry, it is frequently referred to as being composed of three sub-basins. The three sub-basins are depicted in Figure 2, along with their physical measurements.

Estimates of the atmospheric loading of phosphorus to Lake Erie range from 615 metric tons per year (Chapra, 1976) to 1146 metric tons per year (Task Group III, U.S.-Canada Water Quality Agreement, 1978). Both of these estimates were based on extrapolation of measurements made over other lakes. Burns, et al. (1976a) report an estimate of phosphorus input to Lake Erie from shoreline erosion to be 14,800 metric tons per year. This estimate is based on the measurement of the phosphorus content of shoreline materials and of the quantity of shoreline materials eroded. Estimates of phosphorus loading to Lake Erie are presumed to include phosphorus in all of its forms. There is doubt as to what proportion of this total phosphorus would be available for biological uptake. Burns (1976b) suggests that the phosphorus in eroded shoreline materials is not available because of its low solubility. The U.S. Army Corps of Engineers (1975) state that shoreline eroded phosphorus is bound as apatite and not available for entry into the phosphorus cycle of Lake Erie.

16

Future studies will shed more light on this question of availability; however, for the present, the phosphorus contributed from shoreline erosion is excluded from the analysis of biological productivity in Lake Erie.

Basin	(1)	(2)	(3)
Volume	2.27×10^{10} m^3	29.80×10^{10} m^3	15.2×10^{10} m^3
Surface Area	0.311×10^{10} m^2	1.63×10^{10} m^2	0.624×10^{10} m^2
Average Depth	7.3 m	18.28 m	24.36 m
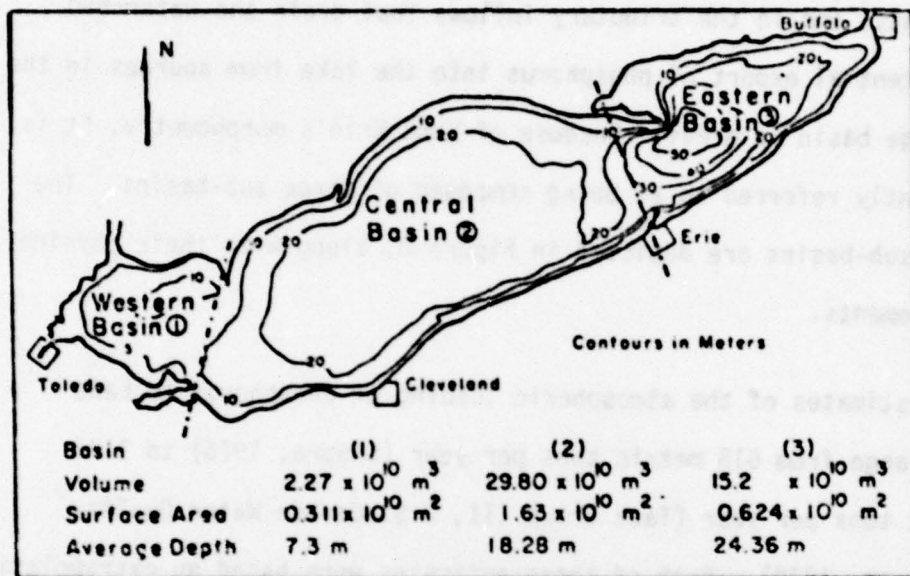
FIGURE 2 Lake Erie Bathymetry and Sub-Basins

There is still a question as to what proportion of the remaining total phosphorus loadings to Lake Erie is available for biological uptake. This results from the lack of detailed budgets of the various forms and lack of knowledge as to what phosphorus forms are available. One recourse is to do a material balance on total phosphorus only.

As part of the re-negotiation of the 1972 Water Quality Agreement between the United States and Canada, "base year" total phosphorus loadings were estimated for the Great Lakes. The purpose of developing "base year" loads was to "smooth out" the non-point contribution to the total load by using historical average data for the tributary

flows and inter-lake exchange flows. The point source and atmospheric loadings used in the development of the "base loads" were reported to be the estimates for the years 1975 and 1976 respectively. The base load data are summarized in Table I.

The U.S. Army Corps of Engineers in their Lake Erie Wastewater Management study have developed estimates of total phosphorus loadings to Lake Erie based on tributary stream flows and an inventory of point source loads. The estimates of stream loadings were based on direct measurement of selected tributaries and establishment of correlations between stream discharge and phosphorus export. The methodology developed was used to develop estimates for total phosphorus loadings to Lake Erie for the years 1970 to 1976. The 1976 estimates and the average of the estimated loads for the period 1970-76 are presented in Table I for comparison with the "base year" estimates.

TABLE I
Estimates* of Total Phosphorus Loadings** to Lake Erie
(in metric tons per year)

	Base Year	1976	1970-76 Average
Western Basin	14,499	12,689	14,208
Central Basin	4,007	4,382	4,163
Eastern Basin	1,463	2,605	1,817
Whole Lake Total	19,969	19,676	20,188

* Estimates developed by the U.S. Army Corps of Engineers Lake Erie Wastewater Management Study

** Shoreline erosion loadings of total phosphorus not included.

Disposition of Phosphorus in Lake Erie

Williams, et al. (1976a) have categorized the forms of phosphorus found in the surficial sediments of Lake Erie as apatite phosphorus, nonapatite inorganic phosphorus, and organic phosphorus. The distribution of sediment phosphorus within the lake did not correlate with locations of major phosphorus inputs. Rather, the distribution was governed by sediment properties such as particle size whose distribution, in turn, is governed by lake bathymetry and lake circulation. The mean value for sediment total phosphorus was found to be 879 mg/kg with a range of 188 mg/kg to 2,863 mg/kg (based on dry sediment). In general, regeneration of phosphorus from the sediments into the water column is not significant under oxic conditions (Williams, et al., 1976b). However, under anoxic conditions the regeneration of phosphorus is significant, having been found to be eleven times as great as under oxic conditions (Burns and Ross, 1972).

Particulate phosphorus constituted 50 to 70 percent of the total phosphorus found in the water column. The western basin averaged 66 percent, the central basin averaged 61 percent, and the eastern basin 56 percent (Burns, 1976c). The quantity of soluble reactive phosphorus varied widely throughout the lake with the western basin and the south shore of the central and eastern basins exhibiting the highest concentrations. The hypolimnetic waters of the central basin were observed to develop increases of soluble reactive phosphorus during anoxia, a direct result of phosphorus regeneration from the sediments. It was also observed that severe wind storms generated sufficiently high bottom currents in certain parts of the lake such

that sediment was physically resuspended. Regeneration of phosphorus accompanied this storm activity, although subsequent sedimentation removed most of that regenerated.

The phosphorus incorporated in the lake biomass varies daily and seasonally. Estimates of this quantity of phosphorus depend upon estimates of the total biomass in the lake as well as knowledge of the percentage of the biomass composed of phosphorus. Vollenweider (1971) cited figures for the mean phosphorus content of phytoplankton ranging from 0.19 percent to 0.5 percent of the dry weight of the phytoplankton biomass. Using the reported molecular formula for phytoplankton cell material (i.e., $C_{106}H_{263}O_{110}N_{16}P$), the computed percentage is 0.87 percent, which is somewhat higher (Richards, et al., 1965). Reported basin wide concentrations of biomass in Lake Erie average about 4 g/m^3 (wet weight) (Munarvar and Munarvar, 1976). Assuming the dry weight to be 25 percent of the wet weight, the dry weight concentration of biomass would be on the order of 1 g/m^3 giving a biomass phosphorus concentration of 0.0087 g/m^3. Based on these very rough estimates, the phosphorus in the biomass could be as great as 50 percent of the total phosphorus measured under certain conditions. Estimates of the annual average concentrations of total phosphorus are given in Table II.

TABLE II

Estimated Annual Average Total Phosphorus
Concentrations in Lake Erie
(in grams per cubic meter)

	Western Basin	Central Basin	Eastern Basin
Base Year and 1976	.0449	.0225	.0239
1970-76* Average	.0393	.0194	.020

* 1970-76 data taken from IJC Great Lakes Water Quality Board
Appendix B, Surveillance Subcommittee Report (1976) except
Eastern Basin (1970, 71, 72) which were taken from Preliminary
Feasibility Report, Vol. 1, U.S. Army Corps of Engineers Lake
Erie Wastewater Management Study, Dec., 1975.

Phosphorus Retention

A simplified concept of the disposition of phosphorus is portrayed
schematically in Figure 3. The phosphorus incorporated in the biomass
is considered, in this scheme, as part of the total phosphorus in the
lake water. The exchanges are indicated by arrows.

This portrayal shows the deposition of phosphorus (primarily
organic detritus) to the sediment as well as release of phosphorus
from the sediment to the lake water (through sorption processes,
bacterial mineralization, and hydrodynamic resuspension). Neglecting
the internal phosphorus exchanges between the biomass and lake water
and assuming the lake volume to be well mixed, the total phosphorus
budget for the scheme depicted in Figure 3 can be written as:

$$\frac{d(CV)}{dt} = M - QC - RM \tag{1}$$

where C = the concentration of total phosphorus in the lake (gm/m^3)

V = lake volume (m^3)

**External Loading
of Phosphorus**

**Phosphorus Exported
in Lake Outflow**

Total Phosphorus
in Biomass

Total Phosphorus
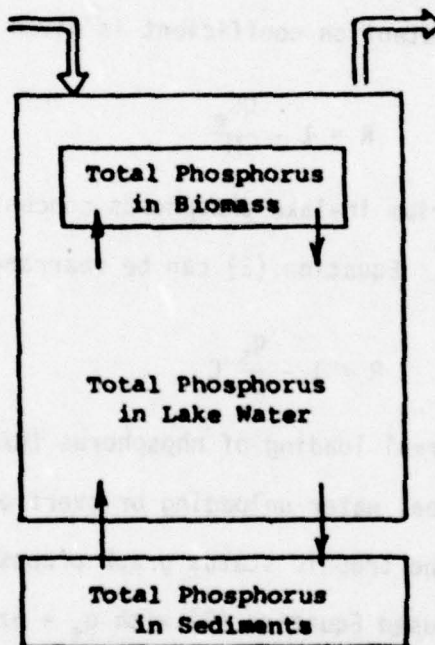in Lake Water

Total Phosphorus
in Sediments

**Figure 3 Schematic Representation of Phosphorus Disposition
in Lake Erie**

M = phosphorus loading to the lake (gm/year)

Q = outflow (m^3/year)

R = retention coefficient which represents the proportion of the total loading of phosphorus that is retained in the lake sediments over a one-year period

t = time (years)

If equilibrium conditions prevail, then there is no change of the in-lake phosphorus concentration over a one-year time period and, from Equation (1), the retention coefficient is given as:

$$R = 1 - \frac{QC_e}{M} \qquad (2)$$

where C_e is the equilibrium in-lake phosphorus concentration and M and Q are also constant. Equation (2) can be rearranged to give:

$$R = 1 - \frac{q_s}{L} C_e \qquad (3)$$

in which L = M/A, the areal loading of phosphorus (g/m^2/yr) and q_s = Q/A, (m/yr) the areal water unloading or overflow rate. Equation (3) was the basis for the trophic status graph proposed by Vollenweider (1973). Dillon (1975) used Equation (3) with q_s = ρz where z is the mean depth and ρ = Q/V, the flushing rate, in a study of lakes in southern Ontario of which some had very high flushing rates. Equation (3) has the desired characteristic of R → 1 as q_s → 0. Similarly, if $q_s C_e$ = L, then R → 0. Both of these limiting conditions are consistent with physical reasoning.

Nutrient Areal Loading Concept

The areal loading concept (i.e., the total nutrient load to the lake over some stated time period divided by the surface area of the lake) is consistent with the fact that the photosynthetic process occurs in the surface waters of a lake where sunlight is present in sufficient intensity for phytoplankton growth (the euphotic zone). The major drawback to the use of the areal loading concept is that it suggests that phosphorus loading to a lake is uniformly distributed over the entire lake surface, a highly unlikely situation. Vollenweider (1971) recognized this drawback as well as the importance of lake water renewal or flushing due to tributary inflows and outflows.

Present-day knowledge of the circulation pattern in Lake Erie indicates that the water that flows from the western basin to the central and eastern basins tends to short-circuit these basins by flowing along the south shore of the lake to the Niagara River outflow. The theoretical ideal renewal time calculated on the basis of dividing the volume of Lake Erie by the Niagara River discharge is less than 3 years. Although the western basin water mass is renewed in less than this time period, the above-mentioned short-circuiting effect suggests that the central and eastern basins may have considerably longer renewal times. Experiments to measure detention times in a physical hydraulic model of Lake Erie revealed that contaminants are flushed from the western basin within months. However, the main water masses of the central and eastern basins are exchanged slowly and long flushing times were observed (on the order of decades) (Rumer, et al., 1976).

Vollenweider presented tentative permissable annual areal loadings of total nitrogen and total phosphorus as a function of lake mean depth. He acknowledged that it would be difficult, in many lakes, to achieve annual loadings less than the recommended permissible loading. The U.S. E.P.A. (National Eutrophication Survey, 1974) also recognizes that nutrient criteria must take into account natural background nutrient levels, land use, lake use and drainage basin characteristics if they are to be realistic and enforceable.

Vollenweider (1971) went on to consider the winter overturn concentrations of total phosphorus that would be representative of the trophic status. He concluded that if early spring-time concentrations were greater than .02 g/m^3, the lake may be condidered to be in danger of eutrophication. Similarly, if the spring time concentrations were less than .01 g/m^3, the lake could be considered to be oligotrophic.

In Figure 4 a plot of the areal phosphorus loading, L, as a function of areal water unloading, q_s, is displayed in the form of a trophic status graph. From Equation (3), it can be seen that the other two parameters, C_e = the equilibrium concentration, and R = the retention coefficient, must be specified in order to classify a particular waterbody according to this scheme. The family of curves depict different combinations of R with the permissable C_e = 0.01 g/m^3 and the dangerous C_e = 0.02 g/m^3. In general, it is seen that according to this scheme higher retention coefficients increase the likelihood of eutrophication. Average values for L for the three basins obtained from Tables I and II are plotted in Figure 4 for reference purposes.
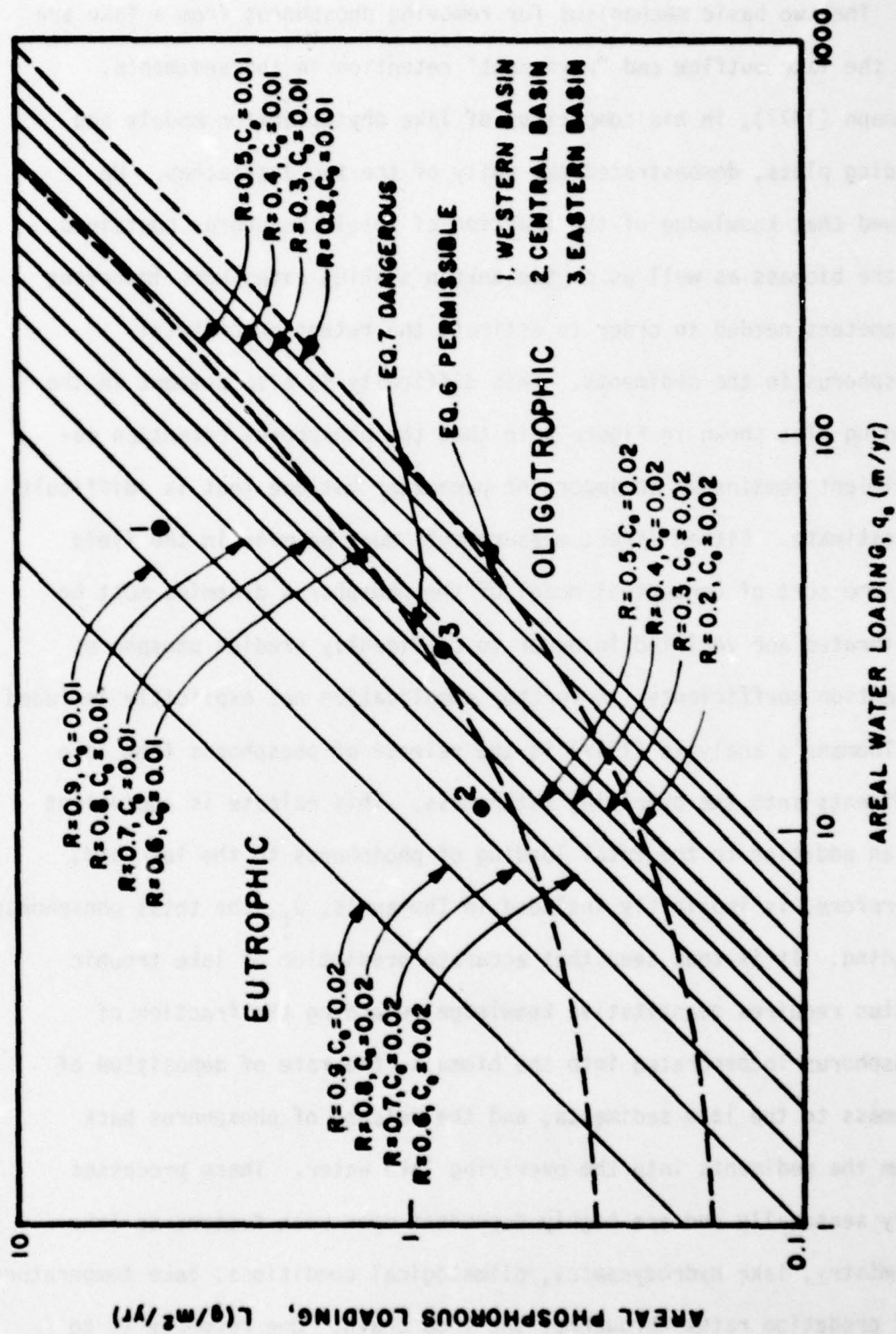
FIGURE 4 TROPHIC STATUS GRAPH FOR LAKES

The two basic mechanisms for removing phosphorus from a lake are via the lake outflow and "permanent" retention in the sediments. Thomann (1977), in his comparison of lake phytoplankton models and loading plots, demonstrated the unity of the two approaches. He showed that knowledge of the fraction of total phosphorus contained in the biomass as well as phytoplankton sinking rates were important parameters needed in order to estimate the retention of total phosphorus in the sediments. This difficulty is also present in the loading plot shown in Figure 4 in that the phosphorus retention co-efficient remains as an important parameter but one that is difficult to estimate. Either direct measurements must be made in the field or some sort of conceptual model of the phosphorus dynamics must be calibrated and verified in order to confidently predict phosphorus retention coefficients. A further complication not explicitly included in Thomann's analysis (1977) is the release of phosphorus from lake sediments into the overlying water mass. This release is equivalent to an addition to the total loading of phosphorus to the lake and, therefore, is implicitly included in Thomann's, J_t, the total phosphorus loading. It is thus seen that accurate prediction of lake trophic status requires quantitative knowledge regarding the fraction of phosphorus incorporated into the biomass, the rate of deposition of biomass to the lake sediments, and the release of phosphorus back from the sediments into the overlying lake water. These processes vary seasonally and are highly dependent upon such factors as lake chemistry, lake hydrodynamics, climatogical conditions, lake temperature, and predation rates throughout the food chain. One recourse is to

develop semi-empirical conceptual models for phosphorus dynamics that incorporate the principal transport processes and which can be calibrated adequately. Such models, when used with a realistic understanding of their limitations, can provide useful estimates of lake trophic status changes resulting from phosphorus load reductions.

Kirchner and Dillon (1975) correlated the phosphorus retention coefficient, R, with areal water unloading, q_s, for several lakes and obtained the following equation:

$$R = 0.426 \exp(-0.271 q_s) + 0.574 \exp(-0.00949 q_s) \qquad (4)$$

Larsen and Mercier (1976), in a similar type of analysis, found the phosphorus retention coefficient to be correlated with either the areal water unloading, q_s, or the flushing rate, $\rho = Q/V$. However, the lakes selected for analysis were primarily oligotrophic.

Chapra (1977) introduced a relationship between the phosphorus retention coefficient and an apparent settling velocity, v, for phytoplankton and the overflow rate, q_s, as follows:

$$R = \frac{v}{v+q_s} \qquad (5)$$

Based on a least squares fit with the data of Dillon and Rigler (1974), Chapra selected a value of v = 16 m/yr. as representative of the Great Lakes. By combining Equations (2) and (5), the following equations for permissible and dangerous areal loading can be obtained:

$$L_{permissible} (g/m^2/yr) = .01 (v + q_s) \qquad (6)$$

$$L_{dangerous} (g/m^2/yr) = .02 (v + q_s) \qquad (7)$$

where $C_e = .01 \ g/m^3$ and $C_e = .02 \ g/m^3$ have been used for permissible and dangerous total phosphorus concentrations, respectively. Equations (6) and (7) are shown in Figure 4 utilizing Chapra's suggested value of $v = 16 \ m/yr$.

The loss of phosphorus to the sediments (i.e., the retained phosphorus) is given by Chapra as $v \ A_s C_e$ where A_s is the sediment surface area and C_e denotes the equilibrium in-lake concentration of total phosphorus. This formulation differs slightly from Thomann's sinking rate, K_s, since his computation is based on the phosphorus incorporated in the biomass, P, as a fraction, α, of the total phosphorus, i.e., $P = \alpha C_e$. The variability of α is not well known, thus precluding the establishment of a single value of α that would relate v and K_s by $\alpha = v/K_s$.

Various estimates of the retention coefficients for Lake Erie are summarized in Table III.

TABLE III

Estimated Phosphorus Retention Coefficients for Lake Erie

Source	Western Basin R_1	Central Basin R_2	Eastern Basin R_3	Whole Lake R
Burns, et al. (1976)	0.72*	0.74	0.31	0.91
Kerchner and Dillon (1975)	0.33	0.54	0.44	0.60
Chapra (1977)	0.22	0.59	0.36	0.69
Base Year**	0.44	0.66	0.22	0.78
1976**	0.36	0.68	0.39	0.81
1970-76 Average**	0.50	0.69	0.32	0.78

*This value is based on an estimate of the loading to the Western Basin that may have been high; thus, R_1 and R may be high also.

** These values were developed utilizing the data from Tables I and II in conjunction with Equation (2).

The variability in the estimated values for the phosphorus retention coefficient for Lake Erie raises a question regarding the assumption of equilibrium conditions. Significant errors are possible if dC/dt is not zero in Equation (1). Because of the relatively low value of V/M for the Western basin, the possible error in computing R is smallest in that basin. Also, the application of the phosphorus retention concept as presented in Equation (2) to the sub-basins of Lake Erie is subject to question because these basins do not behave entirely as separate and completely stirred basins, even over a one-

year time step (particularly the central and eastern basins). There

is occasional exchange of water mass from downstream basin to upstream

basin during seiching and when winds originate from the east. The

uncertainty in developing reliable estimates of in-lake total phosphorus

concentrations and loadings of total phosphorus add to the difficulty

of establishing reliable estimates for the retention coefficients.

PHOSPHORUS MODELING APPROACHES

The objective of controlling phosphorus loading to Lake Erie is not, of course, only to reduce in-lake total phosphorus concentrations below some prescribed value. Rather, the objective is to reduce the high levels of biological productivity in the lake and, thereby, restore the lake to a more desirable trophic status. If the level of biological productivity were to be addressed specifically as the in-lake parameter to be predicted, then a modeling effort would be required that incorporated phytoplankton growth, death, and decay. Such models are developed (DiToro, et al., 1975) and show considerable promise both as frameworks for achieving a better understanding of lake ecosystems and, ultimately, as management tools. These models are based on the conservation of mass principle and utilize available knowledge regarding the kinetics of all reactions that are assumed to take place. In the development of these models, a waterbody is typically divided into segments with the selection of segment size (or volume) dependent upon knowledge of the hydrodynamic behavior of the waterbody and the time resolution desired in the model. In general, the segments are made smaller as more information is known about the details of the lake circulation and to achieve shorter time resolution. Computational time and computer storage limitations generally place a lower bound on segment size since the number of equations to be solved simultaneously greatly increases with smaller segment size and shorter time resolution.

In the formulation of such models, material balance equations may be written for several substances (e.g., phosphorus, nitrogen, dissolved oxygen concentration, etc.). Each of these model compartments will have its own material balance equation and, in general, they must all be solved simultaneously because of mutual interactions (i.e., feed-forward and feedback mechanisms). The basic differential equation for mass conservation can be written as (O'Connor, et al., 1975):

$$V_j \frac{dC_{ij}}{dt} = J_j \pm \Sigma R_{ij} \pm \Sigma W_{ij} \tag{8}$$

where C_{ij} = concentration of the constituent, i, being conserved in segment, j

  i = number of model constituents

  j = number of segments

  V_j = volume of segment j

  J_j = physical transport of the substance through the jth segment (depends on knowledge of the hydrodynamics)

  ΣR_{ij} = the chemical and biological transformations (or reactions) in segment j that involve the substance (or constituent, i) being conserved,

  ΣW_{ij} = the inputs or withdrawals of the substance, i, to or from segment j.

The hydrodynamic transport, J, may be determined from solution to the hydrodynamic equations or may be assigned based on available field data. In general, J is time-dependent and will be significantly modified during the stratification period. Usually, gross simplifications must be made in the representation of J. It should be noted that in the extreme, the internal circulation may be omitted by treating the whole waterbody as one segment (the completely stirred tank reactor

assumption). The degree to which this latter approximation is valid will depend partly on the model time step (i.e., hours, days, months, or years). The number of model compartments is equal to $(i)x(j)$.

DiToro, et al., (1975), have reported on the development of a phytoplankton - zooplankton - nutrient interaction model for western Lake Erie. The western basin (see Figure 2) was broken into seven model segments and seven variables were conserved giving 49 model compartments. The seven model constituents were: chlorophyll a (representative of phytoplankton biomass); organic carbon (representative of zooplankton biomass); organic nitrogen; ammonia nitrogen; nitrate nitrogen; total phosphorus; and inorganic phosphorus. The following model variables were specified for each model segment as a function of time based on available data: water temperature; solar radiation; photoperiod; flows between segments; water clarity; and segment volume. In addition, the following time-dependent boundary conditions were specified: Detriot River inflow; Detroit River chemical quality; Detroit River phytoplankton and zooplankton biomass; Maumee River chemical quality; Maumee River phytoplankton and zooplankton biomass. Twenty kinetic parameters associated with the chemical and biological reactions were specified in advance. The reliability of the model output is limited by the degree to which all of the important parameters have been incorporated in the model formulation; the correctness of the representations for ΣR_{ij}; the availability and adequacy of data for specifying J_i and ΣW_i; and the correctness of the assigned kinetic constants. DiToro, et al. (1975), point out that the determination of a unique set for the kinetic parameters is

beyond present capabilities for a model of this complexity. Hence, the uniqueness of model calibration is not established.

It can be seen that the application of such models requires considerable knowledge of the chemical and biological reactions that determine the dynamics of phytoplankton and zooplankton growth and death and an unusually large data base for model calibration and verification. Although the results of multi-compartment model application have been impressive with notable agreement between model and prototype in some cases, the development of such models for very large waterbodies must still be considered as preliminary. Hornberger, et al. (1975), evaluated a phytoplankton mathematical model by conducting nutrient enrichment experiments in a reservoir near Charlottesville, Virginia. They concluded that the principal application of phytoplankton models is for qualitative determinations and cautioned that each such model must be designed for the specific waterbody under consideration. They found the determination of the "correct" kinetic growth parameters to be the most difficult aspect of model verification. Their calibrated model was used to predict the effects of nutrient enrichment with poor success.

Another approach to the modeling of eutrophication has been the development of mass balance equations for only one nutrient without incorporating the biomass reactions explicitly. This modeling approach greatly reduces the amount of data needed for calibration and the specification of a large set of kinetic parameters. Since diurnal processes do not need to be simulated, the time step for

these models is much greater than that used in the phytoplankton models; hence, they are frequently referred to as long-term nutrient models. Although simplistic in comparison to the sophisticated phytoplankton models, nutrient budget models are based on the same conservation of mass principle. However, the detailed representation of nutrient exchanges and transformations are not attempted. For this modeling approach, Equation (8) can be simplified to:

$$V_j \frac{dC_i}{dt} = J_j \pm \Sigma W_{ij} \tag{9}$$

In the case where only the total mass of the particular element is being conserved, $i \equiv 1$, and the number of model compartments is given by the number of model segments, j.

O'Melia (1972) has discussed this approach to phosphorus modeling and pointed out the importance of sedimentation and the vertical exchange between the hypolimnion and epilimnion in stratified lakes as phosphorus transport processes. His mathematical formulation for the phosphorus budget in a lake did not include exchange at the sediment water interface, although he recognized its existence. In a later paper (Snodgrass and O'Melia, 1975), a more detailed model for phosphorus was presented in which orthophosphate and particulate phosphorus were considered separately. Separate mathematical formulations were presented for the epilimnion and hypolimnion for both forms of phosphorus. Internal loading of phosphorus from the sediments was still not included, although mineralization of organic phosphorus in the hypolimnion by heterotrophic bacteria was included. Sedimentation of particulate phosphorus was accounted for and the

uptake of orthophosphate in biomass production was represented as proportional to the amount of orthophosphate in the epilimnion and was transferred to the particulate phosphorus material balance after conversion. It was suggested that flocculation of particulate phosphorus in the hypolimnion may be important in accelerating the sedimentation process.

Imboden (1974) described a similar two-box two-compartment model for phosphorus which incorporated phosphorus exchange at the sediment-water interface. Through physical reasoning and analysis of the time to achieve steady-state using the model, Imboden concluded that shallow lakes with short detention periods will respond relatively quickly to changes in phosphorus loading. This would be true of the western basin of Lake Erie in which the mean depth is 7.30 m and the detention period is 0.13 year. Conversely, deep lakes will respond very slowly to changes in phosphorus loading. However, phosphorus loading from sediments could alter the expected response of any lake if not properly accounted for, particularly when bottom waters become anaerobic for a significant period or when sediments are resuspended by wave and current action.

Lung, et al. (1976), applied a two-box, two-compartment phosphorus model to a lake in southwest Michigan with a surface area of $1.05 \times 10^4 m^2$, a mean depth of 7.9 m, and a hydraulic detention period on the order of 1-2 months. The model is similar to the one presented by Snodgrass and O'Melia with the exception that the sediment is also modeled as a layered system incorporating both sedimentation of

particulate phosphorus and diffusion of dissolved phosphorus in the
sediments. They found their calibrated model to be a fairly good
predictor of the phosphorus budget when the lake outflow increased
about 30 percent in a subsequent year. This episode was used as a
verification of the model. An unusual feature of this model was the
simulation of particulate and dissolved phosphorus in the first
38 cm below the sediment-water interface. This addition was attempted
because of the availability of sediment phosphorus data.

Chapra (1977) formulated a model for total phosphorus in the
Great Lakes. Each lake was considered as a completely stirred system
with the exception of Lake Erie which was divided into three sub-
basins, each completely mixed. The model time step was one year
and in-lake losses of phosphorus were modeled by a sedimentation
process, $S = vA_sC_e$, where v represented the "apparent" settling
velocity of total phosphorus, A_s was the surface area of the sediments,
and C_e denoted the concentration of total phosphorus in the lake water.
A value of 16 m/yr was selected for the apparent settling velocity.
The total phosphorus level in each basin was simulated historically
for the period 1800 to 1970 using estimated loadings based on available
information for point and diffuse sources. Future conditions were
simulated for hypothetical reductions in phosphorus loading. Chapra
later incorporated phosphorus feedback from the sediments (1978)
and updated has phosphorus loadings based on the Corps of Engineers
measurements (1975). Different values for the "apparent" settling
velocities were obtained for different feedback rates from sediments.
By using two scenarios (F=0 and F≠0), Chapra developed two predictions

which he hoped would bracket the actual response of the lake to changes in total phosphorus loadings.

Lorenzen, et al. (1976), formulated a material balance expression for total phosphorus in Lake Washington which incorporated sedimentation, regeneration of phosphorus from the sediments, and retention of phosphorus in the sediments. Rate constants were determined by calibrating the model for the period 1940 to 1950 and were not changed in the subsequent use of the model for the period 1950 to 1970 during which large changes in loading occurred. The model results were in good agreement with observed concentrations of total phosphorus during this period.

In the Lake Erie Wastewater Management Study (U.S. Army Corps of Engineers, 1975), the Lorenzen, et al. (1976) model was adapted for application to Lake Erie.

LONG-TERM PHOSPHORUS MODEL

Model Description

The LEWMS model was adapted from a simpler version developed by Lorenzen, Smith and Kimmel (1975). It is basically the application of a material balance on total phosphorus incorporating the exchange of phosphorus at the sediment-water interface and the retention of phosphorus in the sediments. Phosphorus loading to the lake basin and phosphorus discharge (or export) from the basin are also accounted for. The lake basin is assumed to be continuously stirred and the effect of the internal lake circulation is presumably integrated out by considering a time step of one year. Lake Erie was simulated using this simple model by treating the Lake as three separate basins arranged in series. Model output provides phosphorus concentrations in the water column and in the sediment interstitial water as a function of time measured in years.

The Lake Erie version of this phosphorus model was utilized as an evaluative tool by the LEWMS staff in order to assess the impact of hypothetical management plans on in-lake phosphorus concentrations. The management plans called for various levels of reduction in the phosphorus loading to the basin.

The application of the model requires the estimation of rate parameters and knowledge of the physical characteristics of the lake basin. One basic assumption made in arriving at the above estimates in the LEWMS study was that the phosphorus concentrations in Lake Erie were at equilibrium with the estimated phosphorus

loadings for the period selected for calibration.

The essential features of the model are depicted in Figure 5. The lake basin is assumed to be completely mixed (the so-called completely stirred tank reactor). In this figure, M represents the annual phosphorus loading to the basin in grams per year (g/yr). C is the phosphorus concentration in the lake water in grams per cubic meter (g/m^3). C_s is the phosphorus concentration in the interstitial water of the sediments in grams per cubic meter. K_1 is the specific rate of phosphorus transfer to the sediments in meters per year (m/yr). The product K_1C represents the annual phosphorus transfer to the sediments in grams per square meter. When this product is multiplied by the sediment surface area in square meters, the total annual phosphorus transfer to the sediments is obtained in grams per year. K_2 is the specific rate of phosphorus release from the sediments in meters per year. The annual phosphorus release per square meter of sediment area is given by the product K_2 and the difference in phosphorus concentration between the interstitial waters and the lake waters. K_3 is the fraction of the total phosphorus input to the sediment which is retained in the sediments and no longer available for exchange. Q denotes the outflow from the basin in cubic meters per year (m^3/yr). Inflow is assumed equal to outflow. The product CQ is the annual export of phosphorus from the basin via the outflow in grams per year.

The material balance equation for phosphorus in the lake water can be written as

$$\frac{dC}{dt} = \frac{M}{V} + \frac{K_2(C_s-C)A}{V} - \frac{K_1CA}{V} - \frac{CQ}{V} \tag{10}$$

where V is the lake basin volume assumed constant in cubic meters and A is the sediment area in square meters. The quantity dC/dt represents the annual change in lake phosphorus concentration required to balance the phosphorus budget. If the basin is at equilibrium and the phosphorus loading is maintained constant, the lake phosphorus concentration remains unchanged and dC/dt equals zero.

The material balance equation for phosphorus in the sediment interstitial water is given by

$$\frac{dC_s}{dt} = -\frac{K_2(C_s-C)A}{V_s} + \frac{K_1CA}{V_s} - \frac{K_1K_3AC}{V_s} \tag{11}$$

where V_s is the interstitial water volume. Again, for equilibrium conditions, the quantity dC_s/dt equals zero.

The above set of coupled equations can be solved for C and C_s as a function of time measured in years. The accuracy of the solution in quantitative terms depends on the adequacy of the model formulation, correctness of the values chosen for the three rate constants (K_1, K_2 and K_3), adequate information on the phosphorus loading rate (M), and data on the lake characteristics (V, V_s, A, and Q). Initial values of C and C_s must also be known. If equilibrium conditions are assumed such that dC/dt = 0, the following two equations are obtained which

relate the three rate constants.

$$K_1 K_3 = \frac{M}{AC_e} - \frac{Q}{A} \qquad (12)$$

$$K_2 = K_1(1-K_3) \, \frac{C_e}{C_{s_e}-C_e} \qquad (13)$$

In this case, C_e and C_{s_e} are the steady-state or equilibrium values for lake and interstitial water concentrations, respectively. With knowledge of any one of the rate constants, the other two are specified by the above equations (12) and (13).

The three basin (or segments) version of this modeling approach for Lake Erie is depicted in Figure 6. In the notation, the first subscript refers to the basin (i.e., C_1 refers to the lake phosphorus concentration in the western basin) and the second subscript denotes the specific rate constant (i.e., K_{12} is the specific rate of phosphorus release, K_2, in the western basin, i.e., basin 1). The physical data for the three basins are given in Table IV. Interstitial water phosphorus concentrations were measured in the central basin in 1975 and an average value for the central basin was given as $C_{s2} = 0.286$ g/m^3. Assuming that interstitial water phosphorus concentrations varied in proportion to the total sediment phosphorus content as reported by the Federal Water Pollution Control Administration (1968), the values for the western and eastern basins were obtained and are given in Table IV.
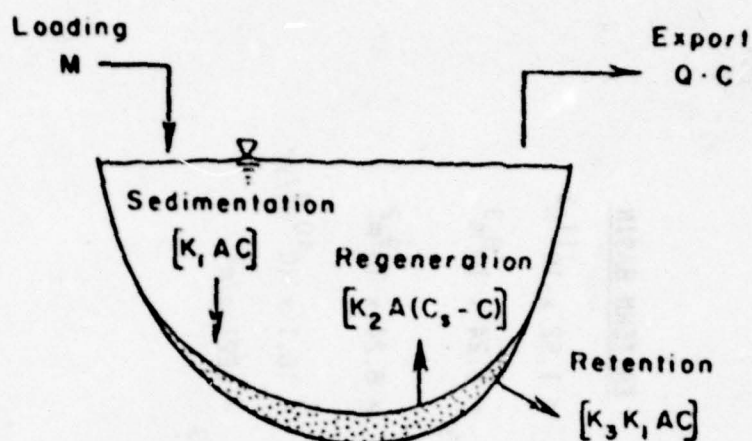
Figure 5   Schematic Representation of Transport Processes Used
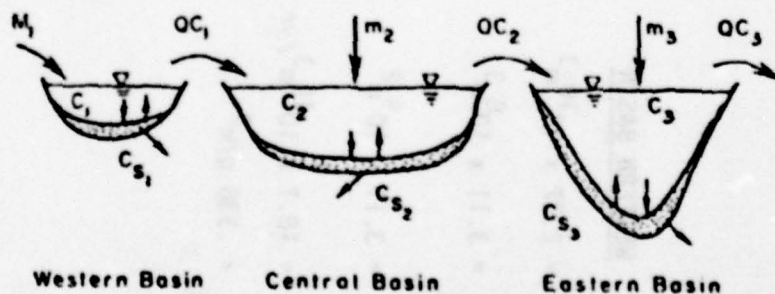in Phosphorus Budget Model (U.S. Army Corps of
Engineers, 1975)

Figure 6   Three Basin Phosphorus Budget Model for Lake Erie
(U.S. Army Corps of Engineers, 1975)

TABLE IV

WESTERN BASIN	CENTRAL BASIN	EASTERN BASIN
$V_1 = 2.27 \times 10^{10} m^3$	$V_2 = 2.98 \times 10^{11} m^3$	$V_3 = 1.52 \times 10^{11} m^3$
$V_{s_1} = 3.11 \times 10^8 m^3$	$V_{s_2} = 1.63 \times 10^9 m^3$	$V_{s_3} = 6.24 \times 10^8 m^3$
$A_1 = 3.11 \times 10^9 m^2$	$A_2 = 1.63 \times 10^{10} m^2$	$A_3 = 6.24 \times 10^9 m^2$
$Q = 18.1 \times 10^{10} m^3/yr$	$Q = 18.1 \times 10^{10} m^3/yr$	$Q = 18.1 \times 10^{10} m^3/yr$
$C_{s_1} = .336 \ g/m^3$	$C_{s_2} = .286 \ g/m^3$	$C_{s_3} = .221 \ g/m^3$

Determination of Model Rate Constants

The rate constants were obtained in the following manner. Based on the findings of Project Hypo (1972), a value of K_{22} for the central basin was determined for both oxic and anoxic conditions. The oxic phosphorus release constant was estimated to be 1.0 m/yr. This value was used for the western basin, the eastern basin, and in the case of the central basin, for 10 months of the year. The anoxic phosphorus release constant was estimated to be 11.0 m/yr. This latter value was assumed to apply in the central basin for 2 months of the year over approximately 26% of the basin area. The annual average values for the phosphorus release constants then become:

$K_{12} = 1.0$ m/yr, $K_{22} = 1.43$ m/yr, and $K_{32} = 1.0$ m/yr.

The total external load of phosphorus to each basin is given by M_i, where $i = 1$, 2, or 3 for the western, central or eastern basin, respectively. The external load for each basin is given by the sum of the basin's tributary area load estimates and the outflow from the upstream basin. Estimates for the phosphorus loadings to the three basins are thus given by the equation $M_i = m_i + QC_{i-1}$, where m_i is the basin tributary area contribution and QC_{i-1} is the flow times the phosphorus concentration in the upstream basin. Estimates for the loads in each of the basins can be obtained from Table I. If it is assumed that equilibrium conditions prevailed during any given year, the other two rate constants, K_1 and K_3 for each basin, can be estimated for that year using Equations (12) and (13).

After the rate constants have been determined, the phosphorus budget model can be utilized to examine the consequences of reduced phosphorus loadings on the lake phosphorus concentrations and interstitial water phosphorus concentrations. The model depicts the transient response of each basin as well as the new equilibrium phosphorus concentrations after a change in external loading. The transient concentrations will have some sensitivity to the rate constants, although they are primarily dependent upon basin volumes and the water discharge.

The above described model would appear to incorporate the principal transport mechanisms for total phosphorus for a lake basin in which exchange of phosphorus with the sediments is important. However, ignoring the internal phosphorus exchange processes represented by the rate constants K_1 and K_2 leads to the mass balance [Equation (2)] for equilibrium conditions (i.e., when $C = C_e$, equilibrium concentration). Using the notation from the above model, the retention coefficient, R, is given as

$$R = \frac{K_3 K_1 A C_e}{M} \tag{14}$$

in which the numerator represents that part of the sedimented total phosphorus which is retained in the sediment matrix.

Equation (12) can be rewritten as follows by letting $K_1 K_3 = v$, the apparent settling velocity (Chapra, 1977), and M/A = L, the areal phosphorus loading.

$$C_e = \frac{L}{v+q_s} \qquad (15)$$

Substitution of Equation (15) into Equation (2) gives the previously obtained Equation (5) (Chapra, 1977), which demonstrates the unity of the two approaches.

$$R = \frac{v}{v+q_s} \qquad (5)$$

At equilibrium, Chapra's (1978) revised model, which includes sediment feedback of phosphorus, is given as

$$M - QC_e - vAC_e + F = 0 \qquad (16)$$

where F denotes the sediment feedback. In this case the retention coefficient, R, is given by,

$$R = \frac{1}{M} [vAC_e - F] \qquad (17)$$

or when F = 0,

$$R = \frac{vAC_e}{M} \qquad (18)$$

Modeling Sensitivity

For purposes of testing the sensitivity of the model output to variations in the lake physical parameters and rate constants, Equations (10) and (11) are written in dimensionless form by introduction of the following dimensionless variables: $C' = C/C_o$, $C_s' = C_s/C_o$, and $t' = \frac{tQ}{v}$. When these are substituted into Equations (10) and (11) we obtain,

$$\frac{dC'}{dt'} = \alpha_1 + \alpha_2(C_s'-C') - \alpha_3 C' - C' \tag{19}$$

$$\alpha_5 \frac{dC_s'}{dt'} = -\alpha_2(C_s'-C') + \alpha_3 C' - \alpha_4 C' \tag{20}$$

in which the coefficients {$\alpha_1 = \frac{M}{C_oQ}$, $\alpha_2 = \frac{K_2A}{Q}$, $\alpha_3 = \frac{K_1A}{Q}$, $\alpha_4 = \frac{K_1K_3A}{Q}$, and $\alpha_5 = \frac{V_s}{V}$} contain all the independent variables. The sensitivity of model output (C' and C_s') to variation in the magnitude of individual independent variables can be more efficiently examined by forming products and quotients of some of the above coefficients (or dimensionless groupings). For instance, it can be seen that,

$$\frac{\alpha_4}{\alpha_1} = \frac{K_3K_1AC_o}{M} = R \tag{21}$$

if C_o is considered as some objective equilibrium concentration {see Equation (14)}. Also, at equilibrium conditions, $\frac{dC'}{dt'} = \frac{dC_s'}{dt'} = 0$, and the model output is independent of $\alpha_5 = V_s/V$, i.e.,

$$\alpha_1 + \alpha_2 (C_s'-1) - \alpha_3 - 1 = 0 \tag{22}$$

$$\alpha_2(C_s'-1) - \alpha_3 + \alpha_4 = 0 \tag{23}$$

where $C' = 1$ since $C = C_o$. Subtracting Equation (23) from Equation (22) gives

$$\alpha_1 - \alpha_4 - 1 = 0 \tag{24}$$

which can also be obtained from Equations (2) and (21).

Since a particular basin has a fixed value of q_s and model
application assumes that the rate constants are independent of time,
it can be seen that the required areal loading, L, is specified
once the objective equilibrium concentration has been established.
We can see that since the rate constants are presumably established
at equilibrium conditions and since α_4 must remain constant thereafter;
it follows that α_1 must also remain constant as well as α_3, α_2 and,
therefore, C_s'. Thus, whatever the ratio of sediment interstitial
phosphorus concentration to in-lake phosphorus concentration used
in establishing the model rate constants, that ratio will remain
unchanged when new equilibrium conditions are obtained after a
change in external loading to the particular lake basin.

Equations (12) and (13), as well as Equation(22) and (23),
establish specific relationships between the three rate constants
(K_1, K_2, K_3) as determined for specific equilibrium values of M,
C, and C_s. It should be noted that for a given basin, the product
K_1K_3 remains unchanged. As discussed earlier, K_1K_3 is equivalent to
Chapra's apparent settling velocity, v, which is the rate at which
phosphorus is taken out of the lake system by retention in the sediments.
It is seen that this parameter is uniquely determined for a given
basin once the equilibrium conditions are defined and the model calibrated.

Probably the best way to portray the sensitivity of the model
output is by plotting equilibrium concentration of inlake total
phosphorus as a function of phosphorus loading to the basin with the
apparent settling velocity, $v = K_1K_3$, as the third parameter. Using
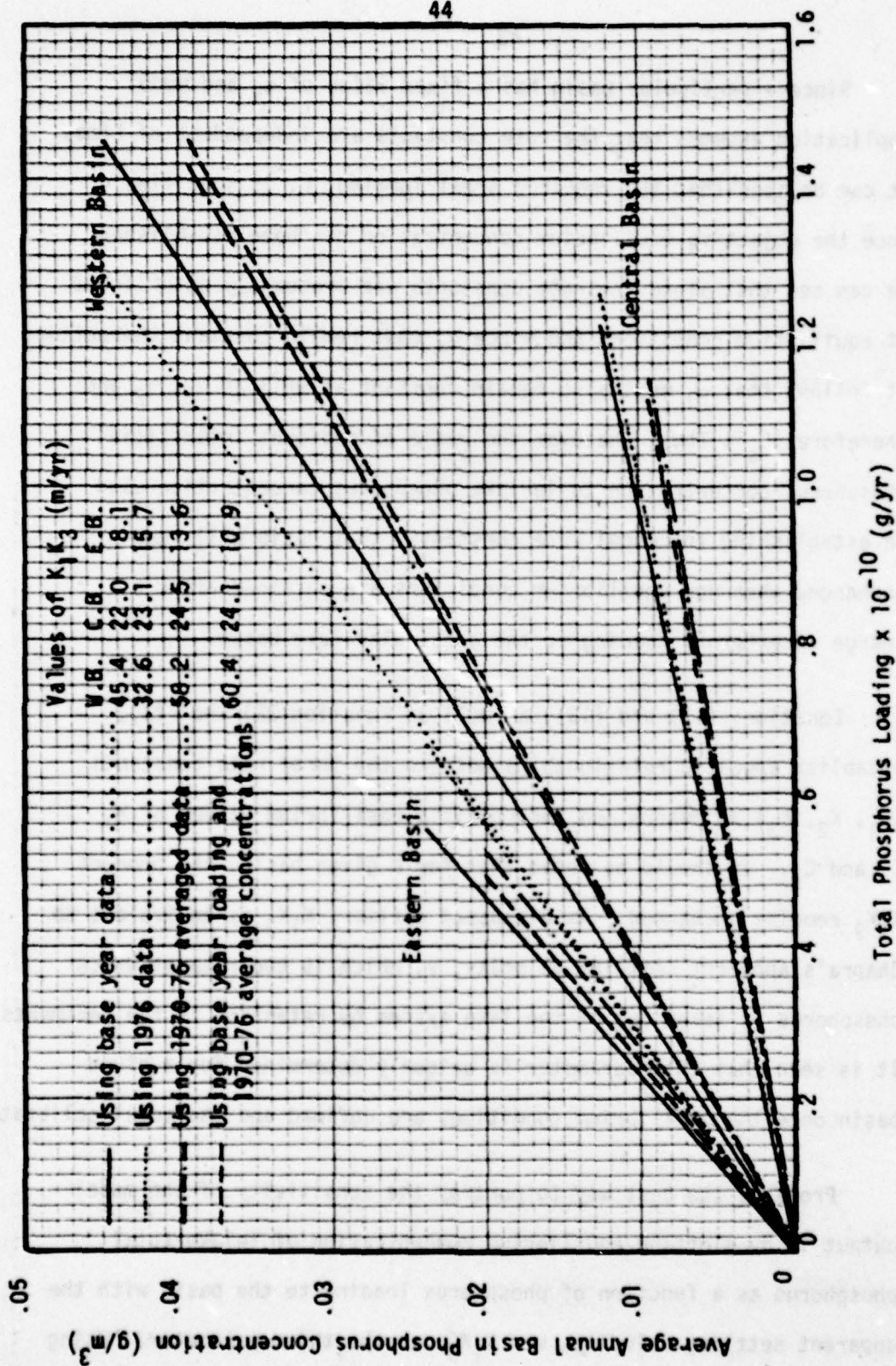
Figure 7 Basin Phosphorus Concentration and Basin Phosphorus Loading for Equilibrium Conditions

the data from Tables I and II, a plot for each of the three sub-
basins is shown in Figure 7.

In summary, the equilibrium phosphorus concentrations calculated
using the model as presently constituted are dependent only on the
so-called retention coefficient, the external loading, and the
water discharge. The retention coefficient is established when the
model rate constants are determined using some assumed equilibrium
conditions. The water discharge from Lake Huron may vary by as much
as 20% depending on hydrologic conditions; however, it is reasonable
to treat it as a constant using the long-term average from the
historical record. Therefore, for practical purposes, the model
output is sensitive only to the assumed equilibrium conditions at
the time of model calibration and will be uniquely prescribed
by the new proposed loadings.

Model Application

It can readily be seen from Figure 7 that the most significant
reduction in phosphorus loading will need to be accomplished in the
western basin of Lake Erie. Since the water flowing from Lake
Huron is adequate in terms of phosphorus concentration level, any
reduction in loading to the western basin would have to occur
through management of the phosphorus originating from the Lake Erie
drainage basin and not from the upper lakes. A reduction in the
concentration of total phosphorus in the western basin leads to a
reduction in the discharge of phosphorus from the western basin to the
central basin and, therefore, from the central to the eastern basin.

The revised loadings and subsequent management requirements within
the drainage basin can all be estimated using the long-term phosphorus
model after in-lake objective total phosphorus concentrations have
been established. Conversely, new equilibrium in-lake concentrations
can be estimated given proposed objective loadings.

The 1972 U.S.-Canada Great Lakes Water Quality Agreement gives
the following objective for phosphorus:

> "concentration should be limited to the extent necessary to
> prevent nuisance growths of algae, weeds and slimes that
> are or may become injurious to any beneficial water use."

The International Joint Commission's Great Lakes Water Quality
Board report of 1974 recommends that this objective be retained and
further states:

> "The existing specific objective is in narrative form because
> the variable response of aquatic organisms dependent in part
> on phosphorus to produce nuisance conditions makes selection
> of a defensible single number very difficult."

Based on the trophic categorization used in this study, this objective
would lead to average total phosphorus concentrations of 0.020 mg/l or
less in each of the Lake's sub-basins. PL 92-500 (1972) calls for
the "rehabilitation and environmental repair" of Lake Erie. Reduction
of in-lake total phosphorus concentrations to 0.02 mg/l or below will
rehabilitate the Lake to a more desirable condition, but the degree
of rehabilitation is not strictly quantifiable. However, when
historical phosphorus loading estimates are compared with current ones
and when the present relative conditions of the three sub-basins are
compared, the reasonable assumption can be made that the central and

eastern basins historically have had lower phosphorus concentrations than the western basin. It was for that reason that the U.S. Army Corps of Engineers (1975) selected the following objective total phosphorus concentrations: western basin, $C_{o_1} = 0.02$ g/m^3; central basin, $C_{o_2} = 0.015$ g/m^3; and eastern basin, $C_{o_3} = 0.015$ g/m^3.

The total phosphorus concentration objectives used for the Lake Erie study were defined because it was necessary at that time to have some quanitiative goal to use in determining required phosphorus load reductions. The Technical Group to Review Phosphorus Loadings (Task Group III, 1978) for the fifth-year review of the Canada-United States Great Lakes Water Quality Agreement has recommended that the total phosphorus loading to Lake Erie be reduced to 11,000 metric tons per year. The rationale behind this recommendation is based on the objective of achieving a 90 percent reduction in the area of anoxia in the central basin of Lake Erie and, thereby, prevent substantial phosphorus release from the sediments. The area of anoxia has been related to phosphorus loadings either by correlations (Chapra, 1978) or directly computed from multi-compartment models that include a compartment for dissolved oxygen (DiToro, et al., 1978). The previously established objective by the U.S. Army Corps of Engineers Lake Erie Wastewater Management Study (1975) of achieving certain levels of in-lake total phosphorus concentrations was based on trophic status correlations (Vollenweider, 1971; Dillon, 1975). It should be noted that none of the above methodologies have been verified by comparing model projections for new conditions resulting from significant changes

in phosphorus loadings with observed new conditions. This step in model development has not been possible.

At this stage, the previously described long-term phosphorus model has been applied to Lake Erie using the recommended total phosphorus loading of 11,000 metric tons as the desired objective. The model constants were determined for two cases: Case A - base year estimated loadings and in-lake phosphorus concentrations; Case B - base year estimated loadings and 1970-76 average in-lake phosphorus concentrations (see Tables I, II and IV). The computed model constants are given in Table V.

There is, at present, a program underway in the Lake Erie drainage basin to reduce the concentration of total phosphorus in the effluents from wastewater treatment facilities to 1 mg/ℓ. When this is accomplished, the estimated annual loading of total phosphorus to Lake Erie will have been reduced to 14,195 metric tons. The estimated distribution of this total load according to source type (point or non-point) and basin is diagrammed schematically in Figure 8. The in-lake concentrations shown in Figure 8 were computed using the model constants given in Table V. It can be seen from Figure 8 that the present ongoing attempts to reduce all point source loads to a concentration of 1 mg/ℓ total phosphorus will not meet the recommended objective of achieving a total phosphorus loading to Lake Erie of 11,000 MT nor does it meet the previously established objective concentrations for all three basins established by the U.S. Army Corps of Engineers Wastewater Management Study.

TABLE V

Computed Model Constants*

	Western Basin		Central Basin		Eastern Basin	
	Case A	Case B	Case A	Case B	Case A	Case B
K_1	51.88 m/yr	68.0 m/yr	38.4 m/yr	43.7 m/yr	16.35 m/yr	20.9 m/yr
K_2	1.0 m/yr	1.0 m/yr	1.4 m/yr	1.4 m/yr	1.0 m/yr	1.0 m/yr
K_3	0.88	0.89	0.57	0.55	0.5	0.52

*Case A - Base Year estimated loadings and 1976 in-lake phosphorus concentrations

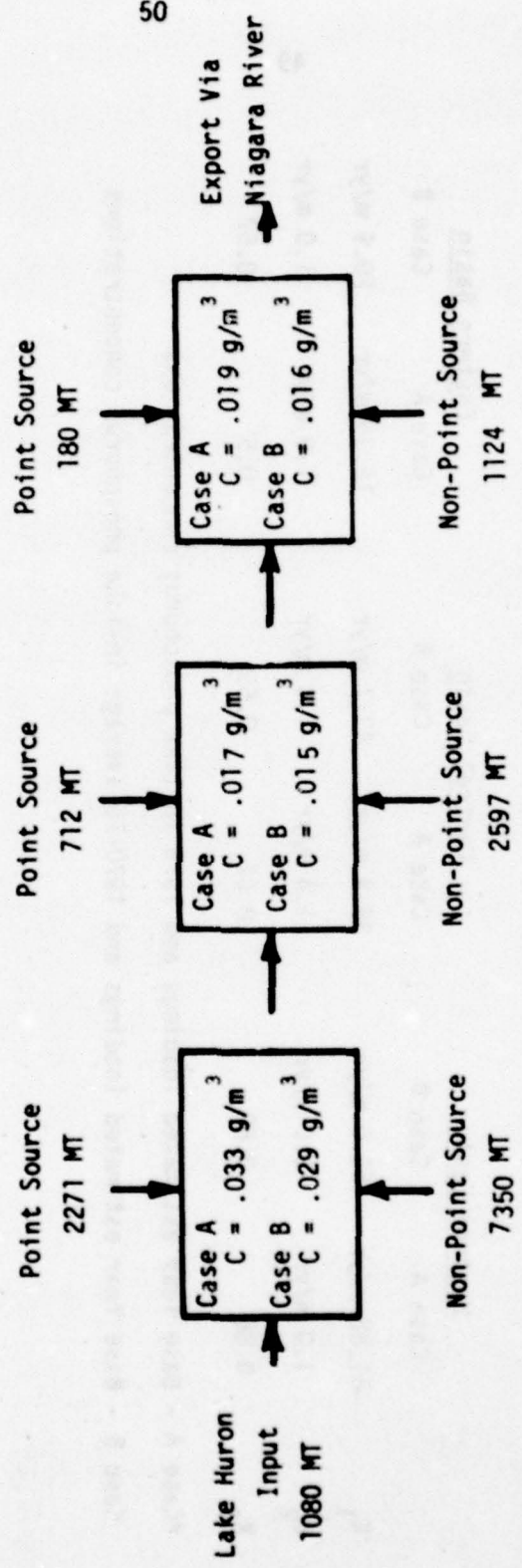Case B - Base Year estimated loadings and 1970-76 average in-lake phosphorus concentrations

Figure 8 Projected In-lake Equilibruim Phosphorus Concentrations using Model Constants (Cases A and B) with all Point Source Effluent Concentrations Reduced To 1 mg/ℓ

In order to further reduce the total loading of phosphorus, management of non-point sources and point sources will be required. The exact trade-off on how this can most economically and effectively be accomplished has not been determined. Two possible scenarios are presented here. If point source loading is not further reduced from the level of 1 mg/ℓ, then the objective of 11,000 MT total load to Lake Erie must be accomplished by reducing non-point source loads by 4314 MT (or a 39% reduction). This scenario is depicted in Figure 9 with the computed in-lake total phosphorus concentrations using model constants from Table V. The scenario portrayed in Figure 9 does meet the recommended total loading of 11,000 MT and the original objective concentrations proposed by the U.S. Army Corps of Engineers Lake Erie Wastewater Management Study (except in the Western Basin).

An alternate scenario would call for all point source effluent concentrations to be further reduced to 0.5 mg/ℓ which would require only 25% reduction in non-point source loading. This scenario is portrayed in Figure 10 with the associated computed in-lake total phosphorus concentrations.

The scenarios depicted in Figures 9 and 10 both achieve the same overall objective of reducing the total phosphorus load to Lake Erie to the recommended value of 11,000 MT. They differ in the degree to which point source loading is reduced relative to non-point source loading. Obviously, there is a continuous spectrum of management scenarios that fall between the two cases presented (and others in which the point sources are assumed to be reduced below 0.5 mg/ℓ).

Lake Huron
Input
1080 MT

Point Source 2271 MT

Case A C = .024 g/m³
Case B C = .021 g/m³

Non-Point Source 4485 MT

Point Source 712 MT

Case A C = .012 g/m³
Case B C = .011 g/m³

Non-Point Source 1585 MT

Point Source 180 MT

Case A C = .013 g/m³
Case B C = .012 g/m³

Non-Point Source 687 MT
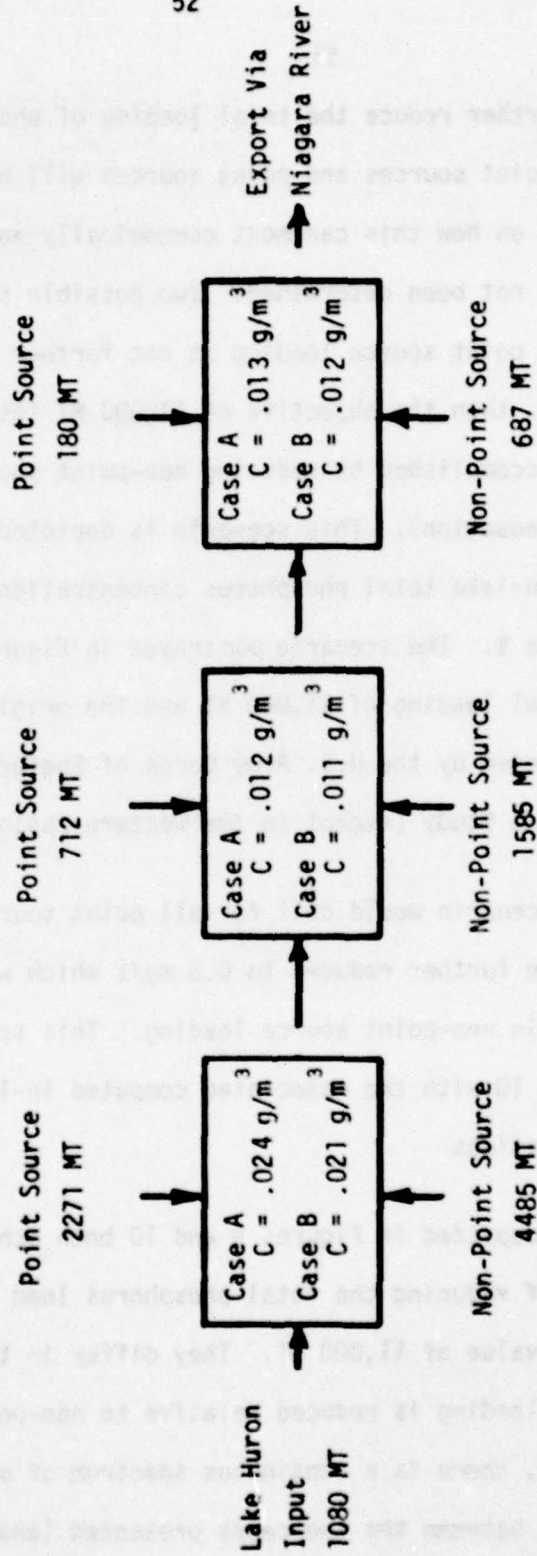
Export Via Niagara River

Figure 9 Projected In-lake Equilibrium Phosphorus Concentrations using Model Constants (Cases A and B) with all Point Source Effluent Concentrations Reduced to 1 mg/ℓ and Non-Point Source Loads Reduced by 39% (Total Load = 11,000 MT)

53

Export Via Niagara River

Point Source 90 MT

Case A
C = .013 g/m³

Case B
C = .012 g/m³

Non-Point Source 847 MT

Point Source 356 MT

Case A
C = .012 g/m³

Case B
C = .011 g/m³

Non-Point Source 1956 MT

Point Source 1135 MT

Case A
C = .024 g/m³

Case B
C = .021 g/m³

Non-Point Source 5536 MT
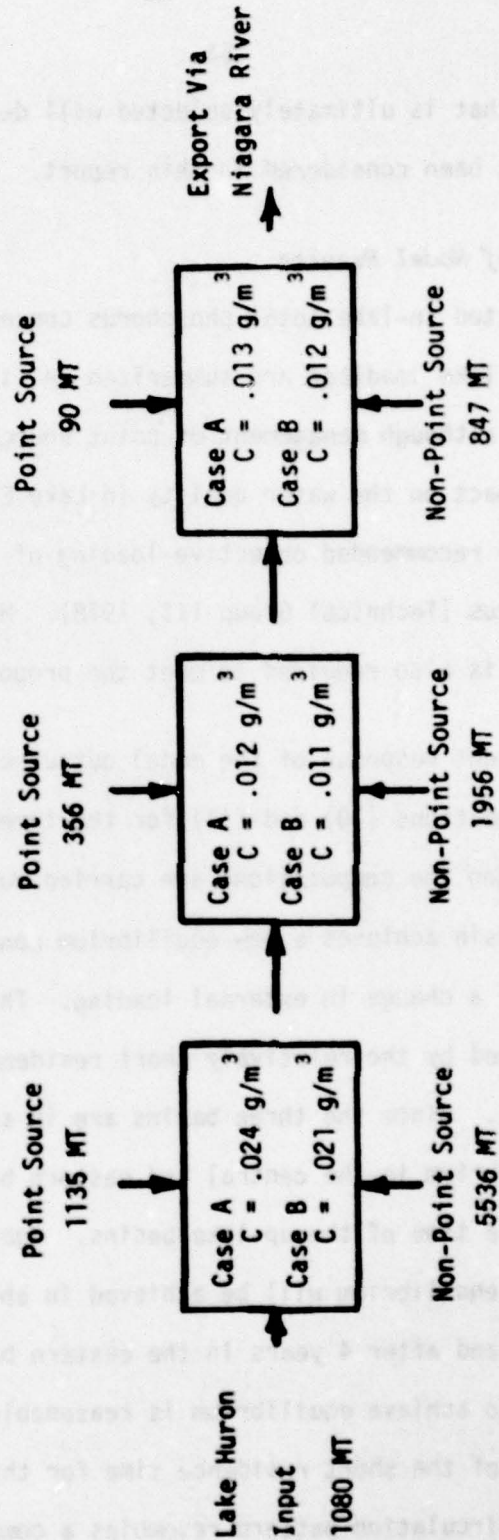
Lake Huron Input 1080 MT

Figure 10 Projected In-lake Equilibrium Phosphorus Concentrations using Model Constants (Cases A and B) with all Point Source Effluent Concentration Reduced to 0.5 mg/ℓ and Non-Point Source Loads Reduced by 25% (Total Load = 11,000 MT)

The strategy that is ultimately selected will depend upon many factors which have not been considered in this report.

Significance of Model Results

The projected in-lake total phosphorus concentrations for the various whole lake loadings are summarized in Figure 11. The findings indicate that although management of point sources will have a beneficial impact on the water quality in Lake Erie, it will not meet the previously recommended objective loading of 11,000 metric tons total phosphorus (Technical Group III, 1978). Management of non-point sources is also required to meet the proposed objectives.

The transient response of the model output can be determined from solution of Equations (10) and (11) for the three-basin version of Lake Erie. When the computations are carried out, it is found that the western basin achieves a new equilibrium concentration within one year after a change in external loading. This rapid response can be explained by the relatively short residence time ($\sim$ 0.13 year) for this basin. Since the three basins are in series, the time to achieve equilibrium in the central and eastern basins is additive to the response time of the up-lake basins. Again, the computations indicate that equilibrium will be achieved in about 3 years in the central basin and after 4 years in the eastern basin. The model output for time to achieve equilibrium is reasonable for the western basin because of the short residence time for this basin and because its internal circulation pattern resembles a completely-stirred tank reactor. Although this assumption is also made for the central
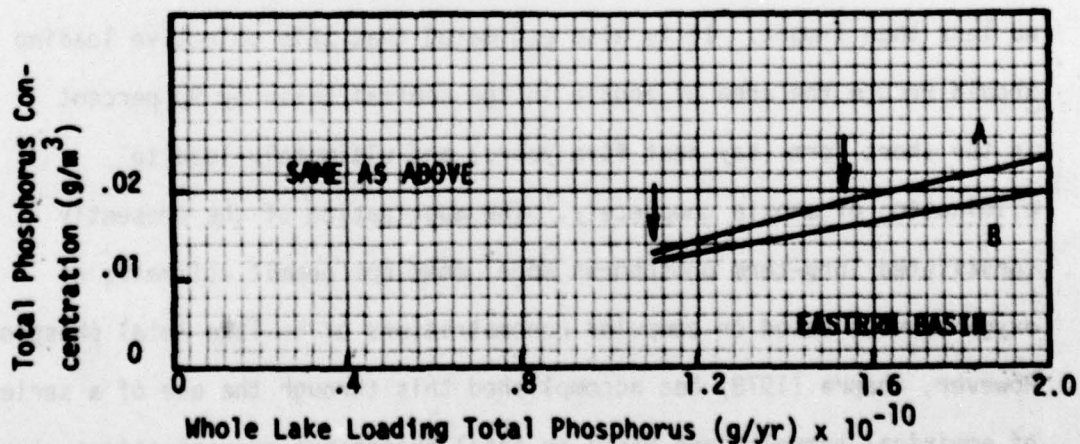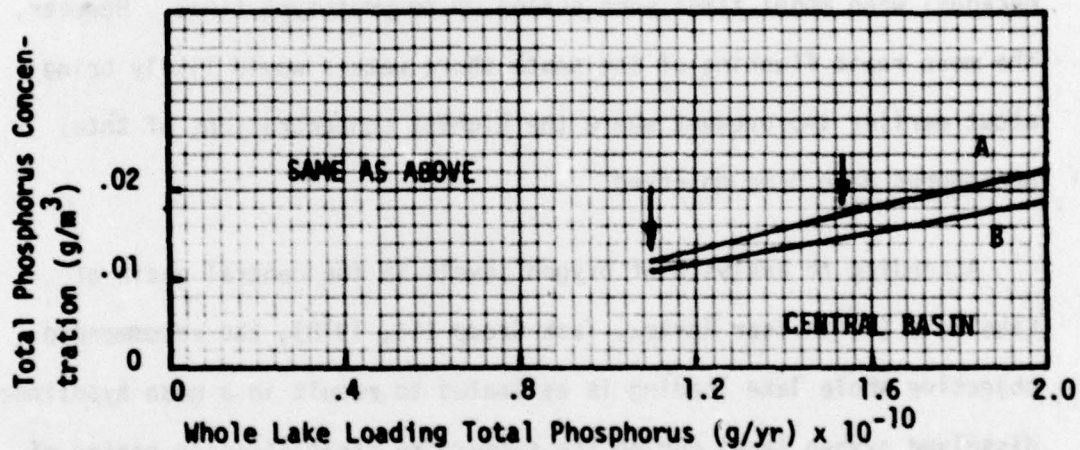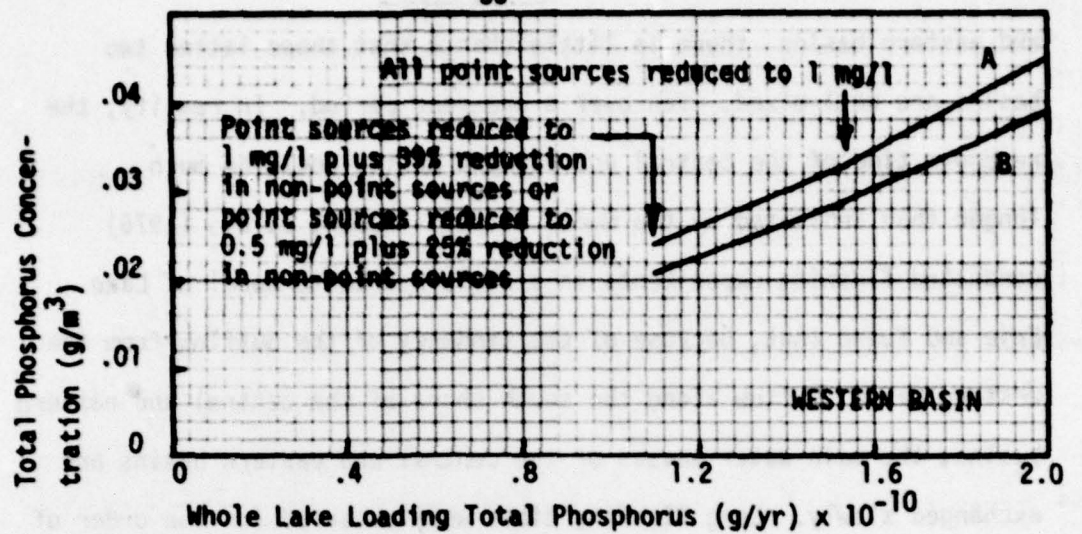
Figure 11   Projected In-Lake Total Phosphorus Concentrations for Indicated Whole Lake Loading Using Base Year Data

Case A =   Model Results Using Base Year Loads and 1976 Concentrations

Case B =   Model Results Using Base Year Loading data and 1970-76 average Concentration data

and eastern basins, there is little chance that these latter two basins are well-mixed, even over a one-year period. In reality, the response time of the central and eastern basins could be much longer than indicated by the model output. Rumer, et al, (1976) conducted flushing experiments in a scale hydraulic model of Lake Erie and found that, because of the tendency of the outflow from the western basin to flow along the south shore of the central and eastern basins, the main water masses of the central and eastern basins are exchanged slowly. Long flushing times were observed (on the order of decades) when model times were scaled up to prototype times. However, the more rapid flushing of the south shore waters would likely bring about earlier improvement where the highest concentrations of total phosphorus have been observed.

According to analysis of oxygen levels in the central basin of Lake Erie (Fifth Year Review, Task Group III, 1978), the recommended objective whole lake loading is estimated to result in a mean hypolimnetic dissolved oxygen level during the summertime stratification period of no less than 1 mg/ℓ. It is also estimated that this objective loading should reduce the area of anoxia in the central basin by 90 percent in the short term (say next five years) and ultimately lead to elimination of anoxia completely. The application of the presently constituted long-term phosphorus model does not permit estimates of oxygen levels based on computed concentrations of in-lake total phosphorus. However, Chapra (1978) has accomplished this through the use of a series of empirical correlations based on total phosphorus concentrations and, second, of primary productivity based on the chlorophyll a

concentrations. He then assumes that the level of dissolved oxygen in the hypolimnion of the central basin is directly proportional to the primary productivity in the surface waters. This procedure enables estimation of dissolved oxygen levels on the basis of computed total phosphorus concentrations. Using the computed in-lake total phosphorus concentration for Case A for the central basin (from Figure 11) with the objective loading at 11,000 metric tons, a miminum dissolved oxygen level of 2.5 mg/ℓ is obtained if Chapra's correlation procedure is followed. Although the correlation of an annual average total phosphorus concentration with a specific summertime level of dissolved oxygen is questionable, the procedure is based on the available data from Lake Erie studies.

Another way to view the significance of the model results is to consider the improvement in trophic status that might result if the recommended objective loading of 11,000 metric tons were obtained. A projection of the possible improvement in trophic status for the new equilibrium situations portrayed in Figure 9 is shown in Figure 12. It can be seen that the projection indicates significant improvement in the central and eastern basins. However, the western basin status remains eutrophic, even though the loading to the western basin has been reduced by 46% through a reduction in point sources to 1 mg/ℓ effluent concentration of total phosphorus and 39% reduction in non-point loading. Assuming the point source phosphorus levels were reduced to 1 mg/ℓ, it would require an 82% reduction in non-point source loading to the western basin in order to bring the trophic status to just between mesotrophic and eutrophic according to the
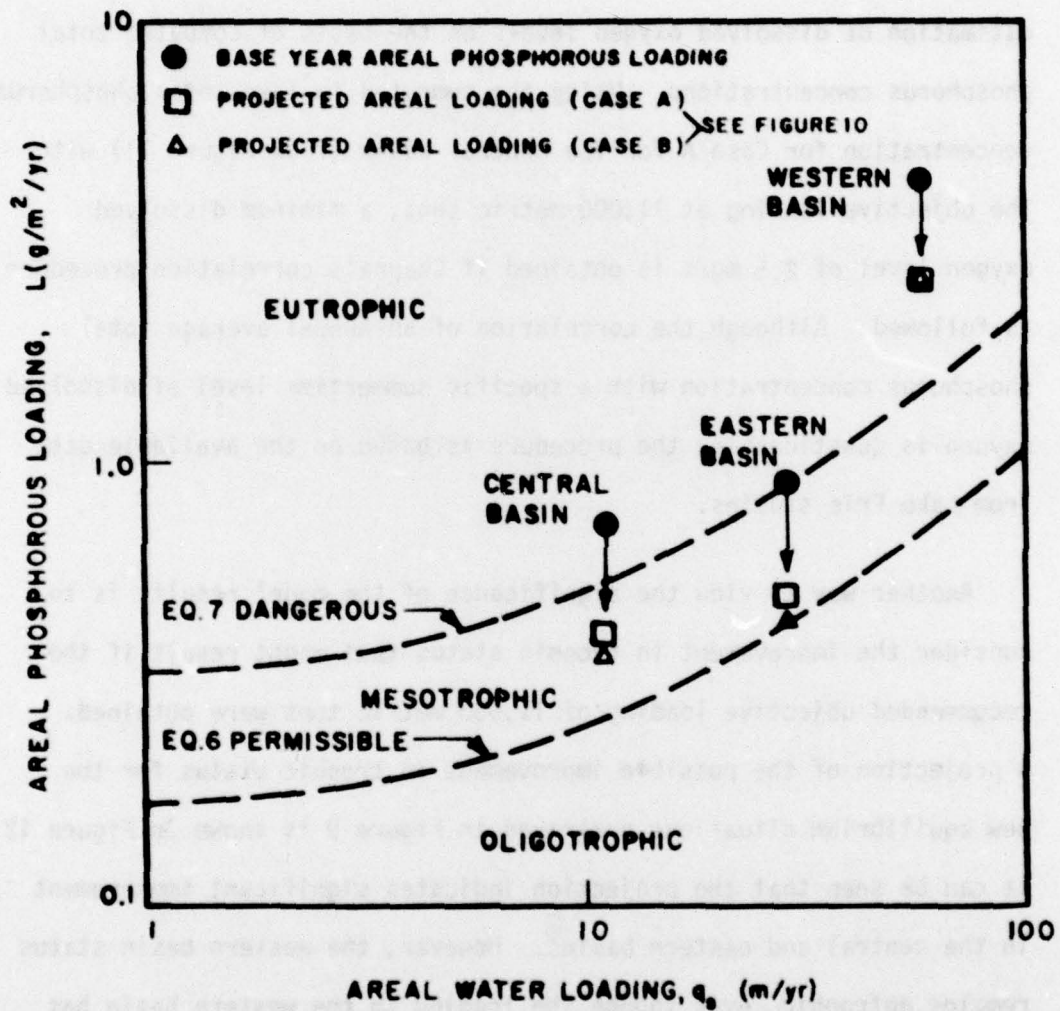
FIGURE 12. TROPHIC STATUS GRAPH (SEE ALSO FIGURE 4)

categorization shown in Figure 12. This amount of reduction in total
phosphorus loading would be very difficult to achieve and probably
unrealistic when considering the small volume of the western basin
compared to the large drainage basin tributary to it.

One final note on the choice of the constants selected for application
of the long-term phosphorus model. They represent crude estimates in
that the calibration of the model was only approximate since all
phosphorus loads have been estimated, in-lake total phosphorus con-
centrations are averages of data which are frequently biased towards
the warmer part of the year, and the assumption of equilibrium has
most likely not been fulfilled. Nevertheless, examination of the
sensitivity of the model output to variations in the model constants
and to the assumed conditions persuades one that the model output
is reasonable (both qualitatively and quantitatively). The degree of
uncertainty in the model projections depends largely on the uncertainty
in the data used for calibration (i.e., the estimates of loadings and
in-lake phosphorus concentrations) and in the rate-of-change of in-
lake conditions (since it is highly unlikely that equilibrium con-
ditions prevail). Chapra (1978) has recently addressed this question
of uncertainty by assigning confidence levels to estimates obtained
from deterministic models (such as the long-term phosphorus model
used in this report).

Although the model output for future equilibrium in-lake phosphorus
concentrations is not dependent upon the phosphorus regeneration rate
constant, K_2, the interstitial phosphorus concentrations are [see

Equations (22) and (23)]. As presently formulated, the higher value of K_2 adopted for the central basin has been justified on the basis of the higher levels of phosphorus release from the sediments during anoxic conditions. However, as in-lake phosphorus concentrations decrease as a result of reductions in phosphorus loading, the length of the period of anoxia and the area of anoxia would also be expected to decrease. The model projections for the recommended objective loading of 11,000 metric tons indicate that, ultimately, no anoxia should occur in the central basin. Under these new conditions, the release of phosphorus from the sediments in the central basin would be more closely approximated by the oxic regeneration rate constant. For this reason, it is expected that future interstitial total phosphorus concentrations may be higher than projected by model output. This would become an important consideration if at some future time, anoxic conditions returned to the central basin.

61

SUMMARY

This report has described the development and application of a
long-term phosphorus model for use as a methodology in evaluating
the in-lake effects that might result from management of phosphorus
loading to Lake Erie. An objective total phosphorus load to Lake
Erie of 11,000 metric tons has been recommended by a task group involved
in the fifth year review of the 1972 Canada-U.S. Great Lakes Water
Quality Agreement. This recommended objective load was used as the
basis for considering possible phosphorus load management strategies
in Lake Erie and the expected in-lake effects that would result. The
findings indicate that if the recommended objective loading of 11,000
metric tons is to be obtained, a reduction in non-point (or diffuse)
sources as well as point sources will be required. The estimated new
equilibrium in-lake concentrations for a loading of 11,000 metric
tons indicate that the trophic status of the central and eastern
basins will become mesotrophic. Also, the excessive biological
productivity and associated periods of anoxia in the central basin
during summertime stratification should be significantly reduced.
Although conditions in the western basin will be improved significantly,
it appears unrealistic to expect that the trophic status of this basin
can be returned to mesotrophic. The large drainage basin, population
pressure and industrial development in the western basin are important
reasons why this would be difficult.

The uncertainty in available data for model application and the
limitations of the model have been discussed. Although the water flow

62

through Lake Erie has been held constant in the analysis, it can be
expected that high flows from Lake Huron would provide greater dilution
and improve in-lake conditions while low flows would serve to worsen
in-lake conditions. The projected estimates of in-lake concentrations
represent basin wide and annual averages. Therefore, the actual
observed concentration of total phosphorus at some position in Lake
Erie is expected to differ from model estimates.

Undoubtedly, the level of uncertainty in projections and estimates
of future in-lake conditions will be reduced as improvements in modeling
capability are made, with better knowledge of the Lake Erie ecosystem,
and as the adequacy of the data base for modeling improves. The present
considerable attention being given to the development of comprehensive
water quality models suggests that improved evaluation methodologies
will soon be available.

REFERENCES

Bader, F.G., Fredrickson, A.G. and H.M. Terichiya (1976), "Dynamics of an Algal-Protozoan Graying Interaction," *Modeling Biochemical Processes in Aquatic Ecosystems*, edited by R.P. Canale, Ann Arbor Science, pp. 257-280.

Beeton, A.M (1969), "Changes in the Environment and Biota of the Great Lakes," *Eutrophication: Causes, Consequences, Correctives*, National Academy of Sciences, Washington, D.C., pp. 150-187.

Bierman, V.J., Jr. and W.L. Richardson (1976), "Mathematical Model of Phytoplankton Growth and Class Succession in Saginaw Bay, Lake Huron," EPA-600/3-76-079, Water Quality Criteria Research of the U.S. E.P.A., Corvallis Environmental Research Laboratory, Corvallis, Oregon.

Brooks, J.L. (1969), "Eutrophication and Changes in the Composition of the Zooplankton," *Eutrophication: Causes, Consequences, Correctives*, National Academy of Sciences, Washington, D.C., pp. 17-28.

Burns, N.M., J.D.H. Williams, J.M. Jaquet, A.L.W. Kemp and D.C.L. Lam, (1976a), "A Phosphorus Budget for Lake Erie," *Journal of the Fisheries Research Board of Canada*, Vol. 33, No. 3, pp. 564-573.

Burns, N.M. (1976b), "Nutrient Budgets for Lake Erie, 1970," *Journal of the Fisheries Research Board of Canada*, Vol. 33, No. 3, pp. 520-573.

Burns, N.M., (1976c), "Temperature, Oxygen, and Nutrient Distribution Patterns in Lake Erie, 1970," *Journal of the Fisheries Research Board of Canada*, Vol. 33, No. 3, pp. 485-511.

Chan, C.H. (1977), "Water Quality Surveys on the Niagara River-1974," Report Series No. 48, Inland Waters Directorate, Water Quality Branch, Burlington, Ontario, Canada.

Chapra, S.C., (1977), "Total Phosphorus Model for the Great Lakes," *Proc. of the American Society of Civil Engineers*, Vol. 103, No. EE2, pp. 147-162.

Chapra, S.C. and S.J. Tarapchak, (1976), "A Chlorophyll a Model and Its Relationship to Phosphorus Loading Plots to Lakes," *Water Resources Research*, Vol. 12, No. 6, pp. 1260-1264.

Chapra, S.C. (1978), personal communication, GLERL, N.O.A.A., Ann Arbor, Mich.

DePinto, J.V., Bierman, V.J. and F.H. Verhoff (1976), "Seasonal Phytoplankton Succession as a Function of Species Competition for Phosphorus and Nitrogen," in *Modeling Biochemical Processes in Aquatic Ecosystems*, edited by R.P. Canale, Ann Arbor Science, pp. 141-170.

64

Dillon, P., (1975), "The Phosphorus Budget of Cameron Lake, Ontario: The Importance of Flushing Rate to the Degree of Eutrophy of Lakes," Limnology and Oceanography, Vol. 20, No. 1, January, pp. 28-39.

Dillon, P., (1975), "The Application of the Phosphorus Loading Concept to Eutrophication Research," Scientific Series No. 46, Canada Centre for Inland Waters, Ontario.

DiToro, D.M., D.J. O'Connor, J.L. Mancini, and R.V. Thomann, (1973), "A Preliminary Phytoplankton-Zooplankton-Nutrient Model of Western Lake Erie," in Systems Analysis and Simulation in Ecology, Vol. 3, 592 pp.

DiToro, D., O'Connor, D., and R. Thomann (1975), "Phytoplankton-Zooplankton-Nutrient Interaction Model for Western Lake Erie," in Systems Analysis and Simulation in Ecology, Vol. III, Academic Press, Inc., New York, N.Y., pp. 423-474.

Freedman, P.L. and R.P. Canale, (1977), "Nutrient Release from Anaerobic Sediments," Proc, American Society of Civil Engineers, Vol. 103, No. EE2, pp. 233-244.

Fry, F.E.J. (1969), "Some Possible Physiological Stresses Induced by Eutrophication," Eutrophication: Causes, Consequences, Correctives, National Academy of Sciences, Washington, D.C., pp. 531-536.

Gakstatter. J.H., Allum, M.O. and J.M. Omernik (1976), "Lake Eutrophication: Results from the National Eutrophication Survey," EPA-600/3-76-079, Water Quality Criteria Research of the U.S. E.P.A., Corvallis Environmental Research Laboratory, Corvallis, Oregon.

Gerloff, G.V. (1969), "Evaluating Nutrient Supplies for the Growth of Aquatic Plants in Natural Waters," Eutrophication: Causes, Consequences, Correctives, National Academy of Sciences, Washington, D.C., pp. 17-28.

"Great Lakes Water Quality Agreement, with Annexes and Texts and Terms of Reference, Between the United States of America and Canada," April 15, 1972.

"Great Lakes Water Quality, 1976, Appendix B. Annual Report of the Surveillance Subcommittee to the Implementation Committee, "Great Lakes Water Quality Board, June, 1977.

Hooper, F.F. (1969), "Eutrophication Indices and Their Relation to Other Indices of Ecosystem Change," Eutrophication: Causes, Consequences, Correctives, National Academy of Sciences, Washington, D.C., pp. 17-28.

Hornberger, G.M., M.G. Kelly, and T.C. Lederman, (1975), "Evaluating a Mathematical Model for Predicting Lake Eutrophication," Bulletin 82, Virginia Water Resource Research Center, Virginia Polytechnic Institute and State University.

Hutchinson, G.E., (1957), A Treatise in Limnology, Vol. 1. John Wiley and Sons, Inc., New York.

Hutchinson, G.E. (1969), "Eutrophication, Past and Present," Eutrophication: Causes, Consequences, Correctives, National Academy of Sciences, Washington, D.C., pp. 17-28.

Imboden, D.M., (1974), "Phosphorus Model for Lake Eutrophication," Limnology and Oceanography, Vol. 19, No. 2, pp. 297-304.

Kirchner, W.B. and P.J. Dillon, (1975), "An Empirical Method of of Estimating the Retention of Phosphorus in Lakes," Water Resources Research, Vol. 11, No. 1, pp. 182-183.

"Lake Erie Environmental Summary: 1963-64," U.S. Department of Interior, Federal Water Pollution Control Administration, 1968.

Larsen, D.P. and H.T. Mercier. (1976), "Shagawa Lake Recovery Characteristics as Depicted by Predictive Modeling," EPA-600/3-76-079, Water Quality Criteria Research of the U.S. E.P.A., Corvallis Environmental Research Laboratory, Corvallis, Oregon.

Larsen, D.P. and H.T. Mercier, (1976), "Phosphorus Retention Capacity of Lakes," J. of the Fisheries Research Board of Canada, Vol. 33 No. 8, pp. 1742-1750.

Livermore, D.F. and W.E. Wunderlich (1969), "Mechanical Removal of Organic Production from Waterways," Eutropication: Causes, Consequences, Correctives, National Academy of Sciences, Washington, D.C., pp. 494-519.

Lorenzen, M., Chen, C.W., Noda, E.K. and L.S. Hwang (1974), "Lake Erie Wastewater Management Study," Tetra Tech Report No. TC 413, Tetra Tech, Inc., Pasadena, California.

Lorenzen, M., D.J. Smith, and L.V. Kemmel, (1976), "A Long-Term Phosphorus Model for Lakes: Application to Lake Washington," In Modeling Biochemical Processes in Aquatic Ecosystems, edited by R.P. Canale, pp. 75-92, Ann Arbor Science, Ann Arbor, Michigan.

Lund, J.W.G. (1969), "Phytoplankton," Eutrophication: Causes, Consequences, Correctives, National Academy of Sciences, Washington, D.C., pp. 306-333.

Lung, W.S., R.P. Canale and P.L. Freedman, (1976), "Phosphorus Models for Eutrophic Lakes," Water Research, Vol. 10, pp. 1101-1114.

Mitchell, R. (1974), _Introduction to Environmental Microbiology_, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

National Eutrophication Survey (1974), "An Approach to a Relative Trophic Index System for Classifying Lakes and Reservoirs," Working Paper No. 24, U.S. E.P.A., Pacific Northwest Environmental Research Laboratory, Corvallis, Oregon.

National Eutrophication Survey (1974), "The Relationships of Phosphorus and Nitrogen to the Trophic State of Northeast and North-Central Lakes and Reservoirs," Working Paper No. 23, U.S. E.P.A., Pacific Northwest Environmental Research Laboratory, Corvallis, Oregon.

Oglesby, R.T. (1969), "Effects of Controlled Nutrient Dilution on the Eutrophication of a Lake," _Eutrophication: Causes, Consequences, Correctives_, National Academy of Sciences, Washington, D.C., pp. 483-493.

O'Melia, C.R., (1972), "An Approach to the Modeling of Lakes," Schweizerische Zeitschrift fur Hydrologie, Vol. 34, pp. 1-33.

"Project Hypo--An Intensive Study of the Lake Erie Central Basin Hypolimnion and Related Surface Water Phenomena," Canada Centre for Inland Waters and U.S. Environmental Protection Agency, Region V, 1972.

Provasoli, L. (1969), "Algal Nutrition and Eutrophication," _Eutrophication: Causes, Consequences, Correctives_, National Academy of Sciences, Washington, D.C., pp. 574-593.

Richards, F.A., J.D. Cline, W.W. Broenkow and L.P. Atkinson, (1965), "Some Consequences of the Decomposition of Organic Matter in Lake Nitinat, An Anoxic Fjord," _Limnology and Oceanography_, Vol, 10, pp. 185-201.

Richardson, W.L. and V.J. Bierman, Jr. (1976), "A Mathematical Model of Pollutant Cause and Effect in Saginaw Bay, Lake Huron," Working Paper No. 23, U.S. E.P.A., Pacific Northwest Environmental Research Laboratory, Corvallis, Oregon.

Rumer, R., K. Kiser, A. Wake and K-H Yu (1976), "Hydraulic Model Study of Lake Erie," Water Resources and Envoronmental Engineering Report No. 76-1, Department of Civil Engineering, State University of New York at Buffalo.

Sager, Paul (1976), "Ecological Aspects of Eutrophication," _Water International_, Vol. 1, No. 2, pp. 11-17.

Snodgrass, W.J. and C.R. O'Melia, (1975), "Predictive Model for Phosphorus in Lakes," _Environmental Science and Technology_, Vol. 9, No. 10, pp. 937-944.

Stumm, W. and E. Stumm-Zollinger (1972), "The Role of Phosphorus in Eutrophication," in Water Pollution Microbiology edited by R. Mitchell, Wiley-Interscience, pp. 11-42.

Task Group III, A Technical Group to Review Phosphorus Loadings, Fifth Year Review of Canada-United States Great Lakes Water Quality Agreement, 22 February, 1978.

Thomann, R.V., R.P. Winfield, and D.M. DiToro, (1974), "Modeling of Phytoplankton in Lake Ontario," Proc. 17th Conf. on Great Lakes Research, International Association for Great Lakes Research, pp. 135-149.

Thomann, R.V., (1977), "Comparison of Lake Phytoplankton Models and Loading Plots," Limnology and Oceanography, Vol. 22, No. 2, pp. 370-373.

United States Army Corps of Engineers, (1975), "Lake Erie Wastewater Management Study, Preliminary Feasibility Report," 3 volumes, Corps of Engineers, Buffalo District, Buffalo, New York.

Vollenweider, R.A., (1971), "Scientific Fundamentals of the Eutrophication of Lakes and Flowing Waters, With Particular Reference to Nitrogen and Phosphorus as Factors in Eutrophication," Organization for Economic Cooperation and Development, Paris, France.

Water Pollution Control Act, Amendments of 1972, Public Law 92-500, 92nd Congress, S. 2770, U.S. Congressional Directory.

Williams, J.D.H., J.M. Jaquet, and R.L. Thomas, (1976a), "Forms of Phosphorus in the Surficial Sediments of Lake Erie," J. of the Fisheries Research Board of Canada, Vol. 33, pp. 413-429.